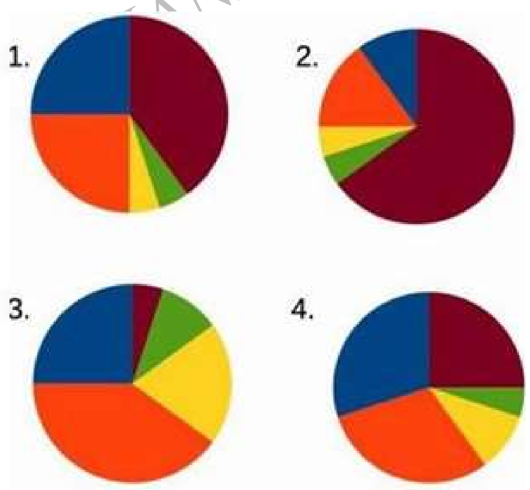
CSIR NET QUESTION PAPER
28-Feb-2025

PART - A:-

Qus 1: If water of pH 8 is diluted 100 times with neutral water (pH = 7) then it will
(1) become acidic
(2) remain basic
(3) become nautral
(4) become heavy

Qus 2: The words TEST, EXAM and EAST are coded as 1382, 2182 and 1937 but not necessarily in that order. How would the word MATE be coded?
(1) 9321 (2) 7321
(3) 7312 (4) 1982

Qus 3: Which of the following pie-charts depicts the distribution of students in the five subjects such that physics and chemistry get equal number of students, 40% of the total go to the life sciences and remaining are equally divided into maths and earth sciences?



(1) 1 (2) 2
(3) 3 (4) 4

Qus 4: How many 4-digit numbers can be generated from the digits 1,2,3,4,5 such that no digit appears more than once, and digit 1 is always somewhere to the left of the digit 2?
(1) 72 (2) 36
(3) 12 (4) 6

Qus 5: In a board meeting of 20 directors, 6 shook everyone else's hands but the remaining 14 did not shake each another's. The total number of handshakes in the meeting was
(1) 26 (2) 84
(3) 99 (4) 190

Qus 6: The average of seven numbers is 71. If we exclude one of these numbers, the average becomes 75. What is that number?
(1) 75 (2) 74
(3) 73 (4) 47

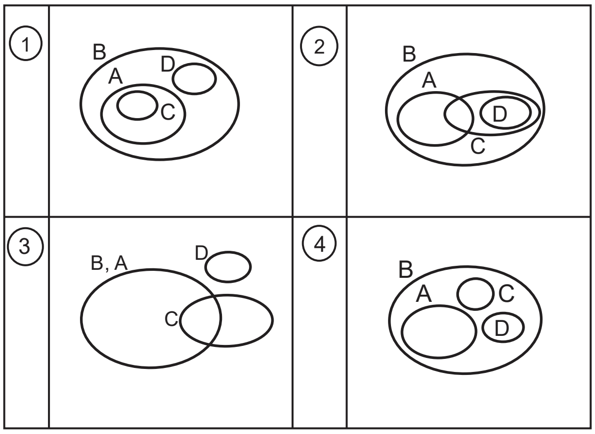
Qus 7: If a map is placed in such a manner that southwest becomes east, then what will north become?
(1) Northeast
(2) Southwest
(3) Northwest
(4) Southeast

Qus 8: Frank, Sam, Tom and David came first, second, third and fourth in a race but not necessarily in this order. Only one had first letter of position matching that of his name. If Tom came first and Sam did not come second then
(1) David came third
(2) Frank came fourth
(3) David came fourth
(4) Sam came fourth

Qus 9: Fifteen distinct points are randomly placed on the circumference of a circle. How many distinct straight lines at the most can be formed by pairs among these points?
(1) 105 (2) 455
(3) 30 (4) 210

Qus 10: Which one of the following Venn diagrams is NOT consistent with the following statements?
All A are B
No D is A

Some A are C



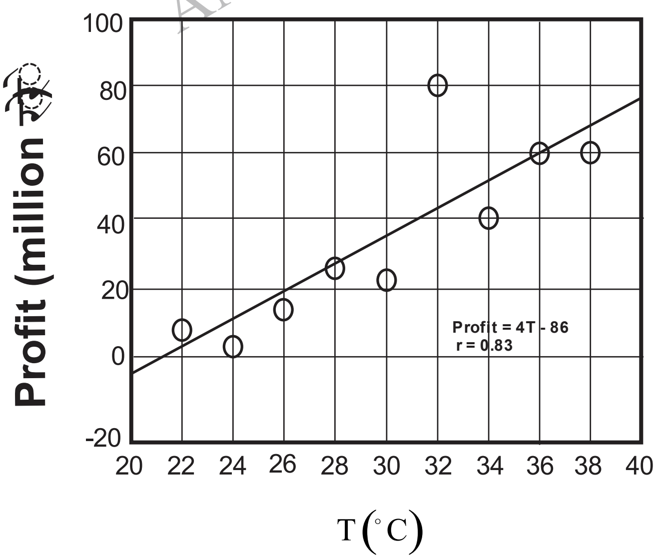
(1) 1 (2) 2
(3) 3 (4) 4

Qus 11: Choose the option to fill in the blank that will make the following statement logically correct?

(1) THE NUMBER OF OCCURRENCES OF THE LETTER "N" IN THIS SENTENCE IS CORRECTLY COUNTED AS ______________

(1) SIX (2) SEVEN
(3) EIGHT (4) NINE

Qus 12: The given figure shows data points and a line fit by least squares method between profit in ice-cream business and mean temperature(T) for a city. Which one of the following inferences can definitely be drawn? (The correlation coefficient r is also given in the figure)



(1) The sum of the squared valued of differences between the observed and expected values of temperature is the minimum.
(2) 83% of the variation in profit is explained by the variation in temperature.
(3) Rise in termperature causes profit to increses.
(4) At $25^{\circ}C$, estimated profit is 14 million Rs.

Qus 13: In the fictional country of Numberia, which of the following porvinces is the odd one out?

(1) SONECON (2) CUGHUSTER
(3) FATWOHUM (4) CAFIVENGUS

Qus 14: A square sheet of 10cm sides is folded along its diagonal to form an isosceles right triangle, and then hypotenuses are folded successively two times to form isosceles right triangles. What is the length of each equal side after the third folding?

(1) 0.625 cm (2) 1.25 cm
(3) 2.5 cm (4) 5 cm

Qus 15: A spherical ball is placed inside a cubic box. If the diameter of the ball is same as the sides of the box, what approximate percentage of volume will be empty?

(1) 12% (2) 24%
(3) 36% (4) 48%

Qus 16: Choose the correct chronological order of the following:-

A: Match B: Trophy C: Toss
D: Result

(1) C,A,D,B (2) A,D,B,C
(3) C,B,A,D (4) D,C,B,A

Qus 17: A water bottle costs Rs 20 that includes cost of the bottle. If the water costs Rs 15 more than the bottle, then what is the cost of the bottle?

(1) Rs 250 (2) Rs 5
(3) Rs 7.50 (4) Rs 10

Qus 18: All those who pass an entrance test take admission into a certain institute. Out of these, some graduate with a degree in 2 years while some fail and are removed, and all graduates from that institute get jobs in the same years.+ In 2022, no one took admission in that institute. Which of the following does not follow necessarily?

(1) No one wrote the entrance test in 2022
(2) No one passed the entrance test in 2022
(3) No one graduated from the institute in 2024
(4) No one got a job from the institute in 2024

Qus 19: In a population of microbial cells, the initial population is 50, and the growth rate is 0.1 per hour. If the population grows exponentially, what will be the approximate size of the population be after 10 hours?

(1) 51 (2) 82
(3) 136 (4) 156

Qus 20: If I walked east 100 metres, turned right and walled 60 meters, turned left and walked 150 meters and turned left again, I would be facing

(1) East (2) North
(3) West (4) South

PART - B:-

Qus 21: Let $X_1, X_2, ..., X_n\left(n \geq 2\right)$ be a random sample from a gamma distribution with shape parameter $\alpha > 0$ and scale parameter $\beta = 1$. For a suitable constant C, the rejection region of the most powerful test for testing $H_0 : \alpha = 1$ against $H_1 : \alpha = 2$ is of the form

(1) $\prod_{i=1}^{n} X_i > C$ (2) $\sum_{i=1}^{n} X_i > C$
(3) $\prod_{i=1}^{n} X_i < C$ (4) $\sum_{i=1}^{n} X_i < C$

Qus 22: Let $f : \mathbb{R} \to \mathbb{R}$ be such that $\sup_{x \neq y} \frac{|f(x) - f(y)|}{|x - y|} = L$, where $1 < L < \infty$. Let $h : \mathbb{R} \to \mathbb{R}$ be a differentiable function satisfying $|h'(x)| \leq \frac{3}{4}$ for all $x \in \mathbb{R}$. For $\alpha > 0$, define $g(x) = \alpha f(x) + h(x)$ for $x \in \mathbb{R}$. Consider the sequence $\{x_k\}_{k=0}^{\infty}$ defined by

$$x_{k+1} = g(x_k), k = 0, 1, ...$$

where $x_0 \in \mathbb{R}$. The sequence $\{x_k\}_{k=0}^{\infty}$ converges to the solution of the equation $x = g(x)$ if

(1) $\alpha < \frac{2}{3L}$ (2) $\alpha < \frac{3}{2L}$
(3) $\alpha < 4L$ (4) $\alpha < \frac{1}{4L}$

Qus 23: For a variable x consider the $\mathbb{R}$ - vector space $V = \{a_0 + a_1 x + a_2 x^2 \mid a_1, a_2, a_3 \in \mathbb{R}\}$

Let $T : V \to V$ be the linear transformation defined by $T(f) = f + \frac{df}{dx}$, where $\frac{df}{dx}$ denotes the derivative of f with respect to x. Which of the following statements is true?

(1) $\left(T^3 - 3T^2 + 3T\right)^{2025}(x) = x$

(2) $\left(T^3 - 3T^2 + 3T\right)^{2025}(x) = x + 1$

(3) $\left(T^3 - 3T^2 + 3T\right)^{2025}(x) = 2025! x$

(4) $\left(T^3 - 3T^2 + 3T\right)^{2025}(x) = 2025! x + 1$

Qus 24: Let $\mathbb{D} = \{z = x + iy \in \mathbb{C} : |z| < 1\}$ be the open unit disc and $f : \mathbb{D} \to \mathbb{C}$ a holomorphic function such that $f(0) = 0$. Let $\psi(z) = |f(z)|^2$ and $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$.

Which of the following statements is False?

(1) f can be extended to $\mathbb{C}$ as an entire function

(2) f must have infinitely many zeros in $\mathbb{D}$

(3) f is not a polynomial

(4) $\exp(f)$ cannot take every complex value

Qus 25: Let $f : \mathbb{C} \to \mathbb{C}$ be the function defined by $f(z) = e^{(\cos(1+i))\sin z}$.

For $z = x + iy \in \mathbb{C}$, write $f(z)$ as $u(x, y) + i\,v(x, y)$, where u, v are real valued functions. Which of the following is the value of $\frac{\partial u}{\partial x}(0, 0)$?

(1) 0 (2) $\left(e+\frac{1}{e}\right)\frac{\cos 1}{2}$

(3) $\left(e-\frac{1}{e}\right)\frac{\cos 1}{2}$ (4) 1

Qus 26: Let $\lambda \in \mathbb{R}$ and $K:[0,1]\times[0,1]\to\mathbb{R}$ be a function such that every solution of the boundary value problem

$$\frac{d^2u}{dx^2}(x)+\lambda u(x)=0;\frac{du}{dx}(0)=u(0),\frac{du}{dx}(1)=0$$

satisfies the integral equation

$$u(x)+\lambda\int_0^1 K(x,t)u(t)\,dt=0.$$

Then

(1) $K(x,t)=\begin{cases}(1+x)(1-t), & 0\le x\le t\le 1\\ (1+t)(1-x), & 0\le t<x\le 1\end{cases}$

(2) $K(x,t)=\begin{cases}-1-x, & 0\le x\le t\le 1\\ -1-t, & 0\le t<x\le 1\end{cases}$

(3) $K(x,t)=\begin{cases}1-x^2, & 0\le x\le t\le 1\\ 1-t^2, & 0\le t<x\le 1\end{cases}$

(4) $K(x,t)=\begin{cases}(1+x)(t-1), & 0\le x\le t\le 1\\ (1+t)(x-1), & 0\le t<x\le 1\end{cases}$

Qus 27: Let U denote the span of $\{e^t, e^{2t}, e^{3t}\}$ in the real vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Consider the $\mathbb{R}$ - vector spaces

$$V=\{f:U\to\mathbb{R} \mid f \text{ is an} \mathbb{R}\text{-linear transformation}\}$$

$$W=\{f\in V \mid f(e^{3t})=0\}$$

Which of the following statements is true?

(1) Both V and W are infinite-dimensional

(2) $\dim V=3$ and $\dim W=1$

(3) $\dim V=3$ and $\dim W=2$

(4) V is infinite-dimensional and $\dim W=0$

Qus 28: Let X,Y and Z be random variables such that $S=\begin{pmatrix}X & Y\\ Y & Z\end{pmatrix}\sim W_2(10,\Sigma)$, where W_2 denotes the Wishart distribution and

$$\Sigma=\begin{pmatrix}1 & \frac{1}{2}\\ \frac{1}{2} & 1\end{pmatrix}.$$

Define $T=Z-\frac{Y^2}{X}$. Then $Var(T)$ equals

(1) $\frac{77}{6}$ (2) $\frac{81}{8}$

(3) $\frac{83}{9}$ (4) $\frac{79}{7}$

Qus 29: Let S denote the set of all solutions of the Euler- Lagrange equation of the variational problem:

minimize $J[y]=\int_0^1\left(y^2+(y')^2\right)dx$

subject to $y(0)=0, y(1)=0, \int_0^1 y^2\,dx=1$.

Then the set $\left\{\varphi\left(\frac{1}{2}\right):\varphi\in S\right\}$ is equal to

(1) $\{-\sqrt{2},\sqrt{2}\}$

(2) $\left\{\frac{\sqrt{2}}{k}:k\in\mathbb{Z},k\ne 0\right\}$

(3) $\left\{\sqrt{\frac{2}{k}}:k\in\mathbb{N}\right\}$

(4) $\{-\sqrt{2},0,\sqrt{2}\}$

Qus 30: Which of the following statements is true?

(1) $\left\{m+ne^{\frac{2\pi i}{3}} \mid m,n\in\mathbb{Z}\right\}$ is a dense subset of $\mathbb{C}$

(2) Open connected subsets of $\mathbb{R}^3$ need not be path-connected

(3) Let X be a topological space and $p:X\to\mathbb{R}$ a continuous surjective open map. If $p^{-1}(\{\alpha\})$ is connected for every $\alpha\in\mathbb{R}$, then X must be connected

(4) Compact subsets of any infinite topological space are closed

Qus 31: Consider the bilinear form $B:\mathbb{R}^4\times\mathbb{R}^4\to\mathbb{R}$ defined by $B(x,y)=x_1y_3+x_2y_4-x_3y_1-x_4y_2$ where $x=(x_1,x_2,x_3,x_4)$ and $y=(y_1,y_2,y_3,y_4)$ in $\mathbb{R}^4$. Let A denote the matrix of B with respect to the standard ordered basis of $\mathbb{R}^4$. Which of the following statements is true?

(1) $\det A=0$

(2) $\det A=-1$

(3) $B(x,x)\neq 0$ for all nonzero $x\in\mathbb{R}^4$

(4) If $x\in\mathbb{R}^4$ is nonzero, then there exists $y\in\mathbb{R}^4$ such that $B(x,y)\neq 0$

Qus 32: Consider the power series

$$\sum_{n=1}^{\infty}\frac{n^{n^2}}{(n+1)^{n^2}}x^n$$

with coefficients in real numbers $\mathbb{R}$. Which of the following statements is true?

(1) The radius of convergence of the series is $\frac{1}{e}$

(2) The series converges at $x=5$

(3) The series converges at $x=3$

(4) The series converges for all x with $|x|<\frac{1}{2}$

Qus 33: We say that a group G has property (A) if every non-trivial homomorphism from G to any group is injective. Which of the following groups has property (A) ?

(1) The cyclic group of order 6

(2) The symmetric group S_5

(3) The alternating group A_5

(4) The dihedral group with ten elements

Qus 34: Let $u=u(x,y)$ be the solution of the Cauchy problem

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=u,\ (x,y)\neq(0,0)$$

$$u(x,1)=\sqrt{1+x^2},\ x\in\mathbb{R}.$$

Then which of the following statements is true?

(1) $u(1,0)=0$

(2) $u(x_1,y_1)=u(x_2,y_2)$ whenever $x_1^2+y_1^2=x_2^2+y_2^2$

(3) $u(1,y)=\sqrt{2}$ for all $y\in\mathbb{R}$

(4) $u(x_1,y_1)=u(x_2,y_2)$ whenever $x_1+y_1=x_2+y_2$

Qus 35: Consider an $M/G/1$ queuing system with arrival rate $\lambda=1$ and independent and identically distributed successive service times having probability density function

$$g(x)=\begin{cases}xe^{-x} & \text{if } x>0\\ 0 & \text{otherwise}\end{cases}$$

Define, for $i=1,2,...$

$$I_i=\begin{cases}1 & \text{if the first transition is from } i \text{ to } i-1\\ 0 & \text{if the first transition is from } i \text{ is } i+1\end{cases}$$

Then, $Var(I_2)$ equals

(1) $\frac{5}{32}$ (2) $\frac{5}{24}$

(3) $\frac{3}{16}$ (4) $\frac{8}{15}$

Qus 36: Let $X_1,X_2,...,X_n\ (n\geq 2)$ be a random sample from Uniform $(\theta,1-\theta)$ distribution, where $-\infty<\theta<\frac{1}{2}$. Then maximum likelihood estimator of θ is

(1) $\min\{1-\min\{X_1,X_2,...,X_n\},\ \max\{X_1,X_2,....,X_n\}\}$

(2) $\max\{1-\min\{X_1,X_2,...,X_n\},\ \max\{X_1,X_2,...,X_n\}\}$

(3) $\min\{\min\{X_1,X_2,...,X_n\},\ 1-\max\{X_1,X_2,...,X_n\}\}$

(4) $\max\left\{\min\left\{X_1, X_2, ..., X_n\right\},\right.$
$\left. 1-\max\left\{X_1, X_2, ..., X_n\right\}\right\}$

Qus 37: Suppose that $X \sim$ binomial $(9,\theta), 0.7, \theta<1$, and $Y \sim$ Poisson $(\lambda), \lambda>0$

If $3E(Y)=E(X)$, then which of the following is true?

(1) $Var(X)>3Var(Y)$

(2) $2Var(Y)<Var(X)<3Var(Y)$

(3) $Var(Y)<Var(X)<2Var(Y)$

(4) $Var(X)<Var(Y)$

Qus 38: $\lim_{n\to\infty}\int_0^1\int_0^1 ...\int_0^1 \frac{x_1^2+...+x_n^2}{x_1+...+x_n}dx_1...dx_n$ equals

(1) 1 (2) $\frac{1}{2}$

(3) 3 (4) $\frac{2}{3}$

Qus 39: Given that $y_1(x)=e^{2x}$ is a solution of the ordinary differential equation (ODE)

$$x\frac{d^2y}{dx^2}-(3+4x)\frac{dy}{dx}+(4x+6)y=0, x>0$$

Let $y_2=y_2(x)$ be the solution of the ODE satisfying the conditions

$$y_2(1)=\frac{e^2}{4}, \frac{dy_2}{dx}(1)=\frac{3e^2}{2}$$

Then which of the following statements is true?

(1) y_2 is a strictly increasing function on $(0,\infty)$

(2) $e^{-2x}y_2(x)\to 1$ as $x\to\infty$

(3) y_2 is a strictly decreasing function on $(0,\infty)$

(4) $e^{-2x}y_2(x)\to 0$ as $x\to\infty$

Qus 40: What is the number of injective functions from $\{1,2,...,7\}$ to $\{1,2,...,10\}$?

(1) 10^7 (2) $\frac{10!}{7!}$

(3) $\frac{10!}{3!}$ (4) 7^{10}

Qus 41: Let $u=u(x,t)$ be a solution of the wave equation $\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}=0, x\in\mathbb{R}, t>0$ satisfying the condition $u(0,t)=0, \forall t\ge 0$. Then which of the following statements is true?

(1) $u(x,t)=0$, whenever $x=t$

(2) $u(x,t)=0$, whenever $x=-t$

(3) $u(-x,t)=u(x,t)$, whenever $x>0, t>0$

(4) $u(-x,t)=-u(x,t)$, whenever $0<x\le t$

Qus 42: Let $\mathbb{H}=\{z=x+iy\in\mathbb{C} \mid y>0\}$ and $f:\mathbb{H}\to\mathbb{C}$ be a non-constant holomorphic function satisfying $|f(z)|<1$ for all $z\in\mathbb{H}$. Which of the following statements is true?

(1) $\lim_{y\to+\infty} f'(iy)=0$

(2) $\lim_{y\to+\infty} f'(iy)$ is a complex number with absolute value 1

(3) $\lim_{y\to+\infty} |f'(iy)|=+\infty$

(4) $\lim_{y\to+\infty} f'(iy)$ is not a real number

Qus 43: Let $X_1, X_2, ..., X_5$ be a random sample of size 5 from an absolutely continuous distribution having median M. Let S denote the number of $X_i's$ greater than 0. For testing $H_0: M=0$ against $H_1: M>0$, let

$$\phi(\underline{X})=\begin{cases}1, & \text{if } S>c\\ v, & \text{if } S=c\\ 0, & \text{if } S<c\end{cases}$$

be a test of size $\alpha=0.05$, where $v\in[0,1]$

and $c \in \{-1, 0, ..., 5\}$ are fixed constants. Then $c+v$ equals

(1) $\frac{11}{25}$ (2) $\frac{49}{6}$

(3) $\frac{103}{25}$ (4) $\frac{53}{6}$

Qus 44: For integers $m, n \geq 1$, let

$I_{m,n} = \frac{1}{2\pi i}\int_C z^m \overline{z}^n dz$, where C is the circle $\{z \in \mathbb{C} : |z| = 1\}$ oriented counterclockwise. Which of the following statements is true?

(1) $I_{m,n} = 1$ if $m = n$

(2) $I_{m,n} = 1$ if $m+1 = n$

(3) $I_{m,n} = 1$ if $m = n+1$

(4) $I_{m,n} = 1$ if $m = n+2$

Qus 45: Suppose ϕ is a most powerful test of size 0.05 for testing a simple null hypothesis H_0 against a simple alternative hypothesis H_1. If the power of the test is 0.4, then which of the following is true?

(1) $(1-\phi)$ is a most powerful test at least 0.6 for testing H_1 against H_0

(2) $(1-\phi)$ is a most powerful test at least 0.4 for testing H_1 against H_0

(3) $(1-\phi)$ is a most powerful test at level 0.05 for testing H_1 against H_0

(4) $(1-\phi)$ is not a most powerful test for testing H_1 against H_0 at any level

Qus 46: For integers $n \geq 0$, let $f_n : [-1, 0] \to \mathbb{R}$ be defined by $f_n(x) = \frac{x}{(1-x)^n}$.

Which of the following statements is true about the series $\sum_{n=0}^{\infty} f_n$?

(1) The series is neither absolutely convergent nor uniformly convergent.

(2) The series is both absolutely convergent and uniformly convergent.

(3) The series is absolutely convergent but not uniformly convergent.

(4) The series is uniformly convergent but not absolutely convergent.

Qus 47: Let $\mathbb{C}[x, y]$ be the polynomial ring in two variables over $\mathbb{C}$. For which of the following ideals I, the quotient ring $\mathbb{C}[x, y]/I$ is not an integral domain?

(1) $I = (x, y)$ (2) $I = (x+y)$

(3) $I = (x^2 + y^2)$ (4) $I = (xy-1)$

Qus 48: Let P be the population proportion of units possessing a certain attribute in a population of N units. Let p be the sample proportion in a simple random sample (without replacement) of n units $(2 \leq n < N)$. Then an unbiased estimator of $P(1-P)$ is

(1) $\frac{(N-n)}{Nn} p(1-p)$

(2) $\frac{(N-n)}{(N-1)^n} p(1-p)$

(3) $\frac{n}{(n-1)} p(1-p)$

(4) $\frac{(N-1)n}{N(n-1)} p(1-p)$

Qus 49: Let $U_1, U_2, ..., U_5$ be 5 urns such that urn U_k contains $2k + k^2$ balls, out of which $2k$ are white balls and k^2 are black balls, $k = 1, 2, ..., 5$. An urn is selected with probability of selecting urn U_k being proportional to $(k+2)$. A ball is chosen randomly from the selected urn. Then, the probability that the urn U_5 was selected, given that the ball drawn is white, is equal to

(1) $\frac{3}{5}$ (2) $\frac{2}{5}$

(3) $\frac{1}{5}$ (4) $\frac{3}{4}$

Qus 50: Consider the sequences $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ defined by $a_n = \frac{e^n + e^{-n}}{2}$ and $b_n = \frac{a_{n+1}}{a_n}$.

Which of the following statements is true?

(1) For each $x \in \mathbb{R}$, there exists an n such that $a_n > x$

(2) For each $x \in \mathbb{R}$, there exists an n such that $a_n < x$

(3) For each $x \in \mathbb{R}$, there exists an n such that $b_n > x$

(4) For each $x \in \mathbb{R}$, there exists an n such that $b_n < x$

Qus 51: Let $u = (a,b,c) \in \mathbb{R}^3$ be a non-zero vector that lies in the orthogonal complement (with respect to the standard inner product) of the row-space of the matrix.

$$A = \begin{pmatrix} 2 & 2 & 7 \\ 3 & 1 & 4 \end{pmatrix}$$

If a,b,c are all integers, then what is the smallest possible value of $|a+b+c|$?

(1) 5 (2) 10

(3) 15 (4) 20

Qus 52: Two blocks of equal mass m are connected by a fiexible inelastic cord of mass M. One block is placed on a smooth horizontal table, the other block hangs over the edge. The total potential energy of the entire cord is given by $\frac{-Mg}{2l}x^2$, where x is the distance of the hanging block from the edge of the table, l is the length of the cord, and g is the gravitational acceleration.

Then

(1) $\ddot{x} = \frac{l}{g}\frac{ml + Mx}{2m + M}$

(2) $\ddot{x} = \frac{l}{g}\frac{Ml + Mx}{m + M}$

(3) $\ddot{x} = \frac{g}{l}\frac{ml + Mx}{m + M}$

(4) $\ddot{x} = \frac{g}{l}\frac{ml + Mx}{2m + M}$

Qus 53: In an examination question paper, all questions are True or False type. These are arranged in such a way that three-fourth of times a question with answer True is followed by a question with answer True. Also two-third of times a question with answer False is followed by a question with answer False. If the question paper has 100 questions, the approximate probability that the correct answer of the 100-th question is True is

(1) $\frac{3}{7}$ (2) $\frac{4}{7}$

(3) $\frac{3}{4}$ (4) $\frac{5}{6}$

Qus 54: Let

$$A = \begin{bmatrix} 0 & a & 0 \\ 0 & 0 & b \\ c & 0 & 0 \end{bmatrix}$$ where a,b,c are real number with $abc = 1$. If $B = A + A^2 + A^3$, then which of the following statements is true?

(1) $\det B = 1$

(2) $\det A = 0$

(3) $\text{rank}(B) = 2$

(4) $\text{rank}\ (B^2) = 1$

Qus 55: Let $f:[0,1] \to \mathbb{R}$ be defined by $f(x) = \sin(x^2)$.

Let $A=\lim_{n\to\infty}\left(\sum_{k=1}^{n} f\left(\frac{k}{n}\right)-n\int_0^1 f(x)\,dx\right)$

Which of the following statements is true?

(1) $A=0$ (2) $A=1$

(3) $A=\frac{\sin(1)}{2}$ (4) $A=\sin\left(\frac{1}{4}\right)$

Qus 56: Let A, B and C be sets. Which of the following sets is equal to $A\setminus(B\setminus C)$?

(1) $A\setminus B$ (2) $(A\setminus B)\cup C$

(3) $A\setminus(B\cup C)$ (4) $(A\setminus B)\cup(A\cap C)$

Qus 57: Consider a multiple linear regression model $Y_i=\beta_0+\beta_1 x_{i1}+\ldots+\beta_p x_{ip}+\in_i$, $1\le i\le n,\ n>(p+1)$ where the errors $\in_i$'s are uncorrelated with zero mean and finite variance $\sigma^2>0$. Here Y_i is the i-th response Let $\widehat{Y}_i$ be the i-th predicted response by the least sqaures estimation method, and let $\widehat{\in}_i=Y_i-\widehat{Y}_i,\ 1\le i\le n$. Then which of the following statements is true?

(1) $Var\left(\widehat{\in}_i\right)\le Var(\in_i),\ 1\le i\le n$

(2) $Cov\left(\widehat{\in}_i,\widehat{\in}_k\right)=Cov(\in_i,\in_k)$ for all $i\ne k=1,2,\ldots,n$

(3) $Var\left(\widehat{Y}_i\right)=Var(Y_i),\ 1\le i\le n$

(4) $E\left(\widehat{Y}_i\right)<E(Y_i),\ 1\le i\le n$

Qus 58: Let V be the $\mathbb{R}$-vector space of 5×5 real matrices. Let $S=\{AB-BA \mid A,B\in V\}$ and W denote the subspace of V spanned by S. Let $T:V\to\mathbb{R}$ be the linear transformation mapping a matrix A to its trace. Which of the following statements is true?

(1) $W=\ker(T)$

(2) $W\subseteq\ker(T)$

(3) $W\cap\ker(T)\subseteq W$

(4) $W\cap\ker(T)\subseteq\ker(T)$

Qus 59: Suppose that the differential equation

$$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+e^{2x}y=0,\ x\in\mathbb{R}$$

transforms into a second order differential equation with constant coefficients under the change of independent variable given by $s=s(x)$ satisfying $\frac{ds}{dx}(0)=1$. Then which of the following statements is true?

(1) $e^{-x}(P(x)+1)$ is a constant function on $\mathbb{R}$

(2) $e^{-2x}P(x)$ is a constant function on $\mathbb{R}$

(3) $s(x)=\frac{e^{2x}}{2},x\in\mathbb{R}$

(4) $P(x)\to 1$ as $x\to\infty$

Qus 60: For integers $n>1$, let $G(n)$ denote the number of groups of order n, up to isomorphism, i.e. $G(n)$ is the number of isomorphic classes of groups of order n. Which of the following statements is true?

(1) If $G(n)=1$, then n is prime

(2) $G(8)=2$

(3) If $\gcd(n,,\varphi(n))>1$, then $G(n)>1$. (Here φ denotes the Euler φ-function)

(4) $\limsup_{n\to\infty} G(n)=2$

PART - C:-

Qus 61: Let $X_1, X_2, ..., X_n\,(n \geq 3)$ be a random sample from a population with absolutely continuous cumulative distribution function $F(\bullet)$. The corresponding order statistics are $X_{1:n} < X_{2:n} < ... < X_{r:n} < ... < X_{n:n}$ For $r = 2,3$ define $Y_{r,n} = nF(X_{r:n})$. Suppose that $Y_{r,n}$ converges in distribution to a random variable Y_r as $n \to \infty, r = 2,3$. Then, which of the following statements are true?

(1) Y_2 follows gamma distribution with $E(Y_2) = 2$

(2) $E(Y_{3,n}) \to 3$ as $n \to \infty$

(3) Y_3 follows beta distribution with $E(Y_3) = \frac{1}{3}$

(4) $Y_{2,n}$ follows beta distribution with parameters 2 and $n-1$

Qus 62: Let $f(x)$ be the polynomial of degree at most 2 that interpolate the data $(-1,2),(0,1)$ and $(1,2)$. If $g(x)$ is a polynomial of degree at most 3 such that $f(x)+g(x)$ interpolates the data $(-1,2),(1,0),(1,2)$, and $(2,17)$ then

(1) $f(5)+g(3) = 50$

(2) $f(5)-f(3) = 4$

(3) $f(1)+g(3) = 50$

(4) $f(5)+g(3) = 74$

Qus 63: Let $f:\mathbb{C} \to \mathbb{C}$ be an entire function such that $f(z) = f(iz)$ for all $z \in \mathbb{C}$. Which of the following statements are true?

(1) $f(z) = f(-z)$ for all $z \in \mathbb{C}$

(2) $f'(0) = f''(0) = f'''(0) = 0$

(3) There is an entire function $g:\mathbb{C} \to \mathbb{C}$ such that $f(z) = g(z^4)$ for all $z \in \mathbb{C}$

(4) f is necessarily a constant function

Qus 64: Let A,B be 2×2 matrices with real entires and $M = AB - BA$. Let I_2 denote the 2×2 identity matrix. Which of the following statements are necessarily true?

(1) If A and B are upper triangular, then M is diagonalizable over $\mathbb{R}$

(2) If A and B are diagonalizable over $\mathbb{R}$, then M is diagonalizable over $\mathbb{R}$

(3) If A and B are diagoanlizable over $\mathbb{R}$, then there exists $\lambda \in \mathbb{R}$ such that $M = \lambda I_2$

(4) There exists $\lambda \in \mathbb{R}$ such that $M^2 = \lambda I_2$

Qus 65: Let Z_1 and Z_2 be two independent discrete random variables such that Z_1 follows binomial distribution with parameters $n = 2$ and $p = \frac{1}{2}$ and Z_2 follows Poisson distribution with mean 1. Consider the following system of equations with three variables x_1, x_2 and x_3.

$$x_1 - 2x_2 + x_3 = 1$$
$$2x_1 - 5x_2 + 2x_3 = 2$$
$$x_1 + 2x_2 + Z_1x_3 = Z_2$$

Then the probability that the given system of equations has infinite number of solutions equals

(1) e^{-1} (2) $\frac{e^{-1}}{2}$

(3) $\frac{e^{-1}}{4}$ (4) $\frac{e^{-1}}{12}$

Qus 66: Let X_1, X_2, X_3 be a random sample of size 3 from an absolutely continuous distribution that is symmetric about 0. For $i = 1,2,3$, let

R_i denote the rank of $|X_i|$ among $|X_1|,|X_2|$ and $|X_3|$. If $T^+ = \sum_{i=1,\, X_i>0}^{3} R_i$ is the Willcoxon signed-rank statistic, then which of the following statements are true?

(1) $P(T^+ = 3) = \frac{1}{4}$

(2) $Var(T^+) = \frac{7}{2}$

(3) $P(T^+ > 3) = \frac{5}{8}$

(4) $P(T^+ > 4) = \frac{1}{8}$

Qus 67: Consider the function $f:\mathbb{R}\to\mathbb{R}$ defined by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Which of the following statements are true?

(1) $\lim_{x\to 0} f(x)$ exists

(2) f is continuous at 0

(3) f is differentiable at 0

(4) $\lim_{x\to 0} f'(x)$ does not exist

Qus 68: For $b\in\mathbb{R}$, let $y_b = y_b(x)$ be the unique solution of the initial value problem $\frac{dy}{dx} = y^5 + y^4 + y^3 + y^2 + y + 1, y(0) = b$ defined on its maximal interval of existence I_b. Then which of the following statements are true?

(1) There exists an $\alpha \in (0,\infty)$ such that for every $b\in\mathbb{R}$ with $b > \alpha$, the solution y_b is bounded above on I_b

(2) There exists an $\alpha \in (0,\infty)$ such that for every $b\in\mathbb{R}$ with $b > \alpha$, the solution y_b is bounded below on I_b

(3) There exists an $\alpha \in (-\infty, 0)$ such that every $b\in\mathbb{R}$ with $b < \alpha$, the solution y_b is bounded above on I_b

(4) There exists an $\alpha \in (-\infty, 0)$ such that for every $b\in\mathbb{R}$ with $b < \alpha$, the solution y_b is bounded below on I_b

Qus 69: Consider the non-homogeneous ordinary differential equation (ODE)

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = \sin\left(e^{-5x}\right),\ x > 0.$$

Then which of the following statements are true?

(1) Every solution of the ODE is bounded on $(0,\infty)$

(2) There exists a solution of the ODE which is unbounded on $(0,\infty)$

(3) Every solution of the ODE is unbounded on $(0,\infty)$

(4) Every solution of the ODE tends to zero as $x\to\infty$

Qus 70: Let (X_1, X_2) be a bivariate normal random vector with $E(X_1) = 1$, $E(X_2) = 0$, $Var(X_1) = 1$, $Var(X_2) = 1$, and correlation coefficient $\frac{1}{2}$. Let U be a $U(0,1)$ random variable, which is independent of (X_1, X_2).

If $Z = \frac{UX_1 + X_2 - U}{\sqrt{U^2 + U + 1}}$, then which of the following statements are true?

(1) The distribution of Z is symmetric about 0

(2) $E(Z^2) = 2$

(3) $Var(Z^2) = 1$

(4) Z and U are independent random variables.

Qus 71: If $\lambda \in \mathbb{R}$ and $p \in \mathbb{R}$ are such that the quadrature formula

$$\int_{x_0}^{x_0+h} f(x)\,dx \approx \lambda h\left(f(x_0)+f(x_0+h)\right)$$

$+ph^3\left(f''(x_0)+f''(x_0+h)\right)$ is exact for all polynomials of degree as high as possible, then

(1) $2\lambda+24p=0$

(2) $7\lambda-12p=4$

(3) $2\lambda+24p=-3$

(4) $7\lambda-12p=11$

Qus 72: Let V be the $\mathbb{R}$-vector space of real valued continuous functions on the interval $[0,\pi]$ with the inner product given by $\langle f,g\rangle=\int_0^\pi f(x)g(x)\,dx$.

Let $S=\left\{\sin(x),\cos(x),\sin^2(x),\cos^2(x)\right\}$ and W be the subspace of V generated by S. Which of the following statements are true?

(1) S is a basis of W

(2) S is an orthonormal basis of W

(3) There exist $f,g\in S$ such that $\langle f,g\rangle=0$

(4) S contains an orthonormal basis of W

Qus 73: Let $X_1,X_2,...,X_n$ be a random sample of size n from $U(\theta,\theta+1)$ distribution, where $\theta\in\mathbb{R}$ is the unknown parameter. If $T_n=\min\{X_1,X_2,...,X_n\}$, $n=1,2,...$ then which of the following statements are true?

(1) T_n is an unbiased estimator of θ

(2) $\lim_{n\to\infty} E_\theta(T_n)=\theta$ for all $\theta\in\mathbb{R}$

(3) T_n is a consistent estimator of θ

(4) $\max\{X_1,X_2,...,X_n\}$ is a consistent estimator of θ

Qus 74: Define

$$S:=\left\{y\in C^1[-1,1]: y(-1)=-1, y(1)=3\right\}$$

Let φ be the extremal of the functional $J:S\to\mathbb{R}$ given by

$$J[y]=\int_{-1}^{1}\left[(y')^3+(y')^2\right]dx$$

Define $\|y\|_\infty := \max_{x\in[-1,1]}|y(x)|$ for every $y\in S$ and let $B_0(\varphi,\varepsilon):=\left\{y\in S:\|y-\varphi\|_\infty<\varepsilon\right\}$,

$B_1(\varphi,\varepsilon):=\left\{y\in S:\|y-\varphi\|_\infty+\|y'-\varphi'\|_\infty<\varepsilon\right\}$.

Then which of the following statements are true?

(1) $\varphi(x)=2x+1$ for every $x\in[-1,1]$

(2) There exists $\varepsilon>0$ such that $J[y]\geq J[\varphi]$ for every $y\in B_0(\varphi,\varepsilon)$

(3) There exists $\varepsilon>0$ such that $J[y]\geq J[\varphi]$ for every $y\in B_1(\varphi,\varepsilon)$

(4) There exists $\varepsilon>0$ such that $J[y]\leq J[\varphi]$ for every $y\in B_1(\varphi,\varepsilon)$

Qus 75: For a variable x, consider the $\mathbb{Q}$- vector space $V=\left\{ax^3+bx^2+cx+d \mid a,b,c,d\in\mathbb{Q}\right\}$ Further, let

$$A=\left\{f:V\to\mathbb{Q} \mid f \text{ is a } \mathbb{Q}-\text{linear transformation}\right\}$$

and $B=\left\{f\in A \mid f(1)=0\right\}$, Which of the following statements are true?

(1) If $f\in B$, then $\dim\ker f=3$

(2) $\dim B=3$

(3) $\dim A=4$

(4) If $f\in A$, then the image of f is a one-dimensional $\mathbb{Q}$-vector space

Qus 76: Let $P(z)$ be a non-constant polynomial over $\mathbb{C}$. Given $R>0$, let

$$S_R=\left\{z\in\mathbb{C}:|P(z)|<R\right\}.$$

Which of the following statements are true?

(1) S_R is an open subset of $\mathbb{C}$

(2) S_R is a bounded subset of $\mathbb{C}$

(3) $|P(z)| = R$ for every z on the boundary of S_R

(4) Every connected component of S_R contains a zero of $P(z)$

Qus 77: Which of the following statements are true?

(1) Let $x, y \in \mathbb{R}$ with $x < y$. Then there exists $r \in \mathbb{Q}$ such that $x < \frac{2^{2024} r}{e} < y$

(2) Let $(a_n)_{n\geq 2}$ be a sequence of positive real numbers. If there exists a positive real number L such that $\limsup_{n\to\infty} \frac{a_n}{\log n} = L$, then $\limsup_{n\to\infty} a_n < \infty$

(3) The set of all finite subsets of $\mathbb{Q}$ is countably infinite

(4) The set of continuous functions from $\mathbb{R}$ to the set $\{0,1\}$ is infinite

Qus 78: Let $X_1, X_2, ..., X_{13}$ be independent and identically distribution (i.i.d) Poisson (θ) random variables, where $\theta > 0$. Then, which of the following statements are true?

(1) $\left(\sum_{i=1}^{8} X_i\right)\left(\sum_{i=8}^{13} X_i\right)$ is an unbiased estimator of $48\theta^2$

(2) $\frac{1}{13}\sum_{i=1}^{13} X_i$ is method of moments estimator of θ

(3) There does not exist any unbiased estimator of $e^{-7\theta}$

(4) $\left(e^{X_7}, \sum_{i=1}^{6} X_i, \sum_{i=8}^{13} X_i\right)$ is a sufficient statistic for θ

Qus 79: Let X be a random variable with probability density function $f(x) = \frac{1}{\pi(1+x^2)}, x \in \mathbb{R}$

If $Z = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}(X)$, then which of the following statements are true?

(1) $E(Z^m) = \frac{1}{m+1}$ for all $m \in \mathbb{N}$

(2) $E(\Phi^{-1}(Z)) = 0$, where $\Phi(\bullet)$ is the cumulative distribution function of standard normal random variable

(3) Z is degenerate at 0

(4) If Z_1 and Z_2 are independent and identically distributed (i.i.d) random variables having distribution same as the distribution of Z, then $Z \overset{d}{=} \frac{Z_1 + Z_2}{2}$

Qus 80: Let $f : [0,1] \to \mathbb{R}$ be a monotonic function. Which of the following statements are true?

(1) f is Riemann integrable on $[0,1]$

(2) The set of discontinuities of f cannot contain a non-empty open set

(3) f is a Lebesgue measurable function

(4) f is a Borel measurable function

Qus 81: Let A,B,C be topological spaces such that A is homeomorphic to B, B is a subspace of C and $\overline{B} = C$. Let C be homeomorphic to a subspaace W of A. Which of the following statements are False?

(1) The space $B, \overline{W}, C$ are homeomorphic

(2) The space B, W, C are homeomorphic

(3) If C is compact, then A,B,C are homeomorphic

(4) If A is connected, then B and C are connected

Qus 82: Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(x) = 0$ for all $x \leq 0$ and for all $x \geq 1$. Define $F(x) = \sum_{n=-\infty}^{\infty} f(x+n), x \in \mathbb{R}$.

Which of the following statements are true?

(1) F is bounded

(2) F is continous on $\mathbb{R}$

(3) F is uniformly continuous on $\mathbb{R}$

(4) F is not uniformly continuous on $\mathbb{R}$

Qus 83: Consider the Markov chain with state space $\{0,1,2\}$ and the transition probability matrix P given by

$$P=\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & 0 & \frac{1}{4} \\ \frac{3}{4} & \frac{1}{4} & 0 \end{pmatrix}$$

Let $P^{(n)}=\left(\left(P_{ij}^{(n)}\right)\right)$ denote the n-step transition probability matrix. Then, which of the following statements are true?

(1) $P_{00}^{(2)}=\frac{3}{4}$

(2) $P_{10}^{(3)}=\frac{39}{64}$

(3) The stationary probability that the chain is in state 2 is $\frac{2}{7}$

(4) State 1 is transient

Qus 84: Consider the function $f:\mathbb{R}^2\to\mathbb{R}$ defined by

$$f(x,y)=\begin{cases} \frac{x^2}{x^4+y^2}+e^{xy}, & (x,y)\neq(0,0) \\ 1, & (x,y)=(0,0) \end{cases}$$

Which of the following statements are true?

(1) f is differentiable on $\mathbb{R}^2\setminus\{(0,0)\}$

(2) All the directional derivatives of f exits at $(0,0)$

(3) f is differentiable on $\mathbb{R}^2$

(4) f is not continuous at $(0,0)$

Qus 85: Define a topology τ on $\mathbb{R}$ as follows: a subset U of $\mathbb{R}$ is in the topology τ if and only if $U=\emptyset$ or $0\in U$. Which of the following statements are true?

(1) The set of all irrational numbers is dense in $(\mathbb{R},\tau)$

(2) For each prime number p, the set $\{0,\sqrt{p}\}$ is dense in $(\mathbb{R},\tau)$

(3) $[0,1]$ is compact at $(\mathbb{R},\tau)$

(4) $(\mathbb{R},\tau)$ is Hausdorff

Qus 86: Let

$$A=\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } B=\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Which of the following statements are true?

(1) Both A and B are diagonalizable over $\mathbb{R}$

(2) A is diagonalizable over $\mathbb{C}$ but not over $\mathbb{R}$

(3) Neither A nor B is diagonalizable over $\mathbb{R}$, but both A and B are diagonalizable over $\mathbb{C}$

(4) Neither A nor B is diagonalizable over $\mathbb{C}$

Qus 87: Let $X_1,X_2,\ldots,X_n$ be a random sample from an exponential distribution with probability density function

$$f(x\mid\theta)=\begin{cases} e^{-(x-\theta)} & \text{if } x\geq\theta \\ 0 & \text{otherwise} \end{cases}$$

where $\theta\in\mathbb{R}$ is an unknown parameter. If $X_{(1)}=\min\{X_1,X_2,\ldots,X_n\}$, then which of the following statements are true?

(1) $\left(X_{(1)}-\frac{2}{n}\ln 5, X_{(1)}\right)$ is a 97% confidence interval for θ

(2) $\left(X_{(1)}-\frac{2}{n}\ln 5, X_{(1)}\right)$ is a 96% confidence interval for θ

(3) $\left(X_{(1)}-\frac{1}{n}\ln 20, X_{(1)}\right)$ is a 95% confidence interval for θ

(4) $\left(X_{(1)}-\frac{1}{n}\ln 20, X_{(1)}\right)$ is a 96% confidence interval for θ

Qus 88: Let X_1, X_2, X_3 be a random sample from Uniform $[0,\theta]$ distribution, $\theta > 0$. Consider the likelihood ratio test of size 0.001 for testing $H_0 : \theta = 3$ against $H_1 : \theta \neq 3$. Then which of the following statements are true?

(1) If $\max\{X_1, X_2, X_3\}$ is 3.1, then H_0 is rejected

(2) If $\max\{X_1, X_2, X_3\}$ is 1.3, then H_0 is rejected

(3) If $\max\{X_1, X_2, X_3\}$ is 0.1, then H_0 is rejected

(4) The power of the test $\theta = 0.3$ is 1

Qus 89: Let $f : \mathbb{C} \setminus \{-1,1\} \to \mathbb{C}$ be a holomorphic function that does not take any value in the set $\{z \in \mathbb{C} : |z-1| < 1\}$. Which of the following statements are true?

(1) f is constant

(2) f has removable signularities at -1 and 1

(3) f is bounded

(4) f has either poles or essential singularities at -1 and 1

Qus 90: A group G is said to be divisible if for every $y \in G$ and for every positive integer n, there exists $x \in G$ such that $x^n = y$. Which of the following groups are divisible?

(1) $\mathbb{Q}$ with ordinary addition

(2) $\mathbb{C} \setminus \{0\}$ with ordinary multiplication

(3) The cyclic group of order 5

(4) The symmetric group S_5

Qus 91: For any $b \in \mathbb{R}$, let $S(b)$ denote the set of all broken extremals with one corner of the variational problem

minimize $J[y] = \int_0^1 (y')^4 - 3\left((y')^2\right) dx$

subject to $y(0) = 0, y(1) = b$

Then which of the following statements are true?

(1) $S(2)$ has exactly two elements

(2) $S\left(\frac{1}{2}\right)$ has exactly one element

(3) $S(2)$ is empty

(4) $S\left(\frac{1}{2}\right)$ has exactly two elements

Qus 92: Consider maximizing the objective function $P = x_1 + x_2$
subject to

$$x_1 + x_2 + 2x_3 \leq 5$$
$$2x_1 - 3x_3 \geq 1$$
$$x_2 + x_3 \leq 0$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Then, which of the following statements are true?

(1) The optimal solution is 4

(2) An optimal points is $(4,1,0)$

(3) The optimal solution is 6

(4) $\left(\frac{1}{2}, 0, 0\right)$ is corner point

Qus 93: If $x = x(t), y = y(t)$ is the solution of the initial value problem

$$\frac{dx}{dt} = x - 4e^{-2t}\, y,$$
$$\frac{dy}{dt} = e^{2t}x - y,$$
$$x(0) = 1, y(0) = 1$$

then which of the following statements are true?

(1) $\lim_{t\to\infty} t^{-2} x(t) y(t) = 0$

(2) $x(1) = 0, y\left(\frac{1}{2}\right) = 0$

(3) $x\left(\frac{1}{2}\right) = 0, y(1) = 0$

(4) $\lim_{t\to\infty} t^{-2} x(t) y(t) = 2$

Qus 94: Let X_1 and X_2 be random variables having absolutely continuous distribution functions. Let $h_i(t)$ denote the hazard function of X_i, $i=1,2$. If $h_1(t)\le h_2(t)$ for all $t\in\mathbb{R}$, then which of the following statements are true?

(1) $P(X_1>1)\ge P(X_2>1)$

(2) $P(X_1>1)\ge P\left(X_2>\frac{1}{2}\right)$

(3) $E(X_1)\ge E(X_2)$ provided both the expectations exist

(4) $h(t)=h_1(t)+h_2(t), t\in\mathbb{R}$ is the hazard function of the random variable $Y=\min\{X_1,X_2\}$

Qus 95: For every integer $n\ge 2$, consider a $\mathbb{C}$-linear transformation $T:\mathbb{C}^n\to\mathbb{C}^n$. Let V be a subspace of $\mathbb{C}^n$ such that $T(V)\subseteq V$. Which of the following statements are necessarily true?

(1) There exists a subspace W of $\mathbb{C}^n$ such that $\mathbb{C}^n=V+W$ and $V\cap W=\{0\}$

(2) There exists a subspace W of $\mathbb{C}^n$ such that $T(W)\subseteq W, \mathbb{C}^n=V+W$ and $V\cap W=\{0\}$

(3) Suppose that there exists a positive integer k such that T^k is the identity map. Then there exists a subspace W of $\mathbb{C}^n$ such that $T(W)\subseteq W, \mathbb{C}^n=V+W$ and $V\cap W=\{0\}$

(4) Suppose that there exists a subspace W of $\mathbb{C}^n$ such that $T(W)\subseteq W, \mathbb{C}^n=V+W$ and $V\cap W=\{0\}$. Then there exists a positive integer k such that T^k is the identity map

Qus 96: Let $u=u(x,t)$ be the solution of the initial-boundary value problem

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, (x,t)\in(0,1)\times(0,\infty)$$

$$u(x,0)=4x(1-x), x\in[0,1]$$

$$u(0,t)=u(1,t)=0, t\ge 0$$

Then which of the following statements are true?

(1) $\lim_{t\to\infty}u(x,t)=0$ for all $x\in(0,1)$

(2) $u(x,t)=u(1-x,t)$ for all $x\in(0,1), t>0$

(3) $\int_0^1 (u(x,t))^2\,dx$ is a non-increasing function of t

(4) $\int_0^1 (u(x,t))^2\,dx$ is a non-decreasing function of t

Qus 97: If u is the solution of the Volterra integral equation

$$u(x)=3+\sin x+\int_0^{\pi}\frac{3+\sin x}{3+\sin t}\,u(t)\,dt$$

then

(1) $u\left(\frac{\pi}{2}\right)=4e^{\frac{\pi}{2}}$

(2) $u(\pi)=3e^{\pi}$

(3) $u(-\pi)=4e^{-\pi}$

(4) $u\left(-\frac{\pi}{2}\right)=4e^{\pi}$

Qus 98: Consider the polynomial $f(x)=x^{2025}-1$ over $\mathbb{F}_5$, where $\mathbb{F}_5$ is the field with five elements. Let S be the set of all roots of f in an algebraic closure of the field $\mathbb{F}_5$. Which of the following statements are true?

(1) S is a cyclic group

(2) S has $\varphi(2025)$ elements, where φ denotes the Euler φ-function

(3) S has $\varphi(2025)$ generators, where φ denotes the Euler φ-function

(4) S has 81 elements

Qus 99: For a 4×4 positive definite real symmetric matrix A and real numbers a,b,c,d, consider the 5×5 matrix

$$B=\left(\begin{array}{c|cccc} 0 & a & b & c & d \\ \hline a & & & & \\ b & & & & \\ c & & A & & \\ d & & & & \end{array}\right)$$

Which of the following statements are necessarily true?

(1) $\det(B)>0$ for every nonzero $(a,b,c,d)\in\mathbb{R}^4$

(2) $\det(B)>0$ for infinitely many $(a,b,c,d)\in\mathbb{R}^4$

(4) $\det(B)\le 0$ for every $(a,b,c,d)\in\mathbb{R}^4$

(4) $\det(B)\le 0$ for infinitely many $(a,b,c,d)\in\mathbb{R}^4$

Qus 100: For each positive integer n, define $f_n:[0,1]\to\mathbb{R}$ by $f_n(x)=nx(1-x)^n$

Which of the following statements are true?

(1) $(f_n)_{n\ge1}$ does not converge pointwise on $[0,1]$

(2) $(f_n)_{n\ge1}$ converges pointwise to a continuous function on $[0,1]$

(3) $(f_n)_{n\ge1}$ converges pointwise to a discontinuous function on $[0,1]$

(4) $(f_n)_{n\ge1}$ does not converge uniformly on $[0,1]$

Qus 101: Which of the following statements are true?

(1) The value of the Euler φ-function is even for all integers $n\ge3$

(2) Let G be a finite group and S a subset of G with $|S|>\frac{|G|}{2}$. Then $\{ab:a,b\in S\}=G$

(3) The polynomial ring $\mathbb{R}[x_1,\ldots,x_n]$ is Euclidean domain for all integers $n\ge1$

(4) The subset $\{f\in C[0,1]:f(1/2)=0\}$ of the ring $C([0,1])$ of continuous functions from $[0,1]$ to $\mathbb{R}$ is a prime ideal

Qus 102: Let $f\in\mathbb{R}[x]$ be a product of distinct monic irreducible polynomials $P_1,P_2,\ldots,P_n$, where $n\ge2$. Let (f) denote the ideal generated by f in the ring $\mathbb{R}[x]$. Which of the following statements are true?

(1) $\mathbb{R}[x]/(f)$ is a field

(2) $\mathbb{R}[x]/(f)$ is a finite dimensional $\mathbb{R}$-vector space

(3) $\mathbb{R}[x]/(f)$ is a direct sum of fields, each of which is isomorphic to $\mathbb{R}$ or $\mathbb{C}$

(4) There are no non-zero elements $u\in\mathbb{R}[x]/(f)$ such that $u^m=0$ for some $m\ge1$

Qus 103: Let $g:\mathbb{R}\to\mathbb{R}$ be a continuous function. Define $f(x)=\int_0^x(x-t)g(t)\,dt,\ x\in\mathbb{R}$

Which of the following statements are true?

(1) $f(0)=0$

(2) $f'(0)$ exists and $f'(0)=0$

(3) $f''(0)$ exists and $f''(0)=g(0)$

(4) $f''(0)$ exists but $f''(0)\ne g(0)$

Qus 104: Let $u=u(x,y)$ be the solution of the boundary value problem

$$\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0,\ (x,y)\in(0,1)\times(0,1)$$
$$u(x,0)=e^{\pi x},u(x,1)=-e^{\pi x},x\in[0,1]$$
$$u(0,y)=\cos(\pi y)+\sin(\pi y),y\in[0,1]$$
$$u(1,y)=e^{\pi}\left(\cos(\pi y)+\sin(\pi y)\right),y\in[0,1]$$

Then there exists a point $(x_0,y_0)\in(0,1)\times(0,1)$ such that

(1) $u(x_0,y_0)=\sqrt{2}\,e^{\pi}$

(2) $u(x_0,y_0)=e^{\pi}$

(3) $u(x_0, y_0) = -1$

(4) $u(x_0, y_0) = -e^{\pi}$

Qus 105: Suppose that a sequence of random variables $\{X_n\}_{n\geq 1}$ and the random variable X are defined on the same probability space. Then which of the following statements are true?

(1) X_n converges to X almost surely as $n \to \infty$ implies that X_n converges to X is probability as $n \to \infty$

(2) X_n converges to X in probability as $n \to \infty$ implies that X_n converges to X almost surely as $n \to \infty$

(3) If $\sum_{n=1}^{\infty} \mathbb{P}\left[|X_n - X| > \delta\right] < \infty$ for all $\delta > 0$, then X_n converges to X almost surely as $n \to \infty$

(4) If X_n converges to X in distribution as $n \to \infty$, and X is a constant with probability 1, then X_n converges to X in probability as $n \to \infty$

Qus 106: Consider the system of two particles with total kinetic energy

$$T = \frac{5}{2}\dot{x}^2 + l^2\dot{\theta}^2 + 2l\dot{x}\dot{\theta}\cos\theta$$

and Lagrangian

$$L = \frac{5}{2}\dot{x}^2 + l^2\dot{\theta}^2 + 2l\dot{x}\dot{\theta}\cos\theta + 2gl\cos\theta,$$

where x, θ are generalized coordinates, and g, l are positive constants. Then the non-zero frequency of the normal mode of the system with small oscillations $(|\theta| \ll 1)$ is

(1) $\frac{5}{3}\sqrt{\frac{g}{l}}$ (2) $\sqrt{\frac{5g}{3l}}$

(3) $\frac{5}{2}\sqrt{\frac{g}{l}}$ (4) $\sqrt{\frac{5g}{2l}}$

Qus 107: Consider the linear regression model $Y = X\beta + \in$, with r regressors and an intercept. Random error $\in \sim N_n(0, \sigma^2 I_n)$ and X has full column rank. Here I_n denotes the identity matrix of order n. Regression coefficients are estimated by the least squares estimation method. Let $\hat{\sigma}^2$ and $\hat{\sigma}^2_{MLE}$, respectively, be the mean squares residuals and the maximum likelihood estimator of σ^2. Then, which of the following statements are true?

(1) $MSE\left(\hat{\sigma}^2_{MLE}\right) < MSE\left(\hat{\sigma}^2\right)$ if $r = 2, n = 12$

(2) $Var\left(\hat{\sigma}^2_{MLE}\right) > Var\left(\hat{\sigma}^2\right)$ if $2 \leq r \leq n-2, n \geq 12$

(3) $Var\left(\hat{\sigma}^2_{MLE}\right) < Var\left(\hat{\sigma}^2\right)$ if $1 \leq r \leq n-2, n \geq 3$

(4) $MSE\left(\hat{\sigma}^2_{MLE}\right) > MSE\left(\hat{\sigma}^2\right)$ if $r = 7, n = 12$

Qus 108: Let $(a_n)_{n\geq 1}, (b_n)_{n\geq 1}$ and $(c_n)_{n\geq 1}$ be sequences given by

$a_n = (-1)^n(1 + e^{-n})$,

$b_n = \max\{a_1, ..., a_n\}$, and

$c_n = \min\{a_1, ..., a_n\}$

Which of the following statements are true?

(1) $(a_n)_{n\geq 1}$ does not converge

(2) $\limsup_{n\to\infty} a_n = \lim_{n\to\infty} b_n$

(3) $\liminf_{n\to\infty} a_n = \lim_{n\to\infty} c_n$

(4) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} c_n$

Qus 109: Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta, 1)$ distribution where $\theta \in \mathbb{R}$ is unknown. Let δ_n be the Bayes estimator of θ, under the sqaured error loss function $L(\theta, a) = (a - \theta)^2, a, \theta \in \mathbb{R}$ and the prior distribution $N(1, 2)$.

If $\frac{1}{\sqrt{n}}\left[(2n+1)\delta_n - 1 - 2n\theta\right]$ converges in distribution to a random variable Z, as $n\to\infty$, then which of the following statements are true?

(1) $\delta_n \xrightarrow{P} \theta$, as $n\to\infty$, for all $\theta\in\mathbb{R}$

(2) Z follows normal distribution

(3) $E\left(Z^4\right)=48$

(4) $E\left(Z^2\right)=1$

Qus 110: For a variable x, consider the real vector space $V=\left\{ax^3+bx^2+cx+d \mid a,b,c,d\in\mathbb{R}\right\}$

Let $D:V\to V$ be the linear transformation where $D(f)$ is the derivative of f with respect to x, and $M:V\to V$ be the linear transformation $M(f)=xD(f)$. Which of the following statements are true?

(1) $DM\neq MD$

(2) $D+M$ is invertible

(3) DM is invertible

(4) rank $(DM)=$ rank (MD)

Qus 111: Consider a linear model

$Y_i=\beta_1+\beta_2+\ldots+\beta_i+\in_i, 1\le i\le n$

where error $\in_i$'s are uncorrelated with zero mean and finite variance $\sigma^2>0$. Let $\widehat{\beta_i}$ be the best linear unbiased estimator (BLUE) of $\beta_i, i=1,2,\ldots,n$. Then which of the following statements are true?

(1) The sum of squares residuals is strictly positive with probability 1.

(2) For every $\beta_i, 1\le i\le n$, there are infinitely many linear unbiased estimators

(3) $Var\left(\widehat{\beta_1}\right)=\sigma^2$

(4) Y_3-Y_2 is the BLUE of β_3

Qus 112: For a positive real number $a, \sqrt{a}$ denotes the positive square root of a. Consider the function $f:\mathbb{R}^2\to\mathbb{R}$ defined by

$$f(x)=\begin{cases}\frac{x}{|x|}\sqrt{x^2+y^2} & , x\neq 0\\ 0 & , x=0\end{cases}$$

Which of the following statements are true?

(1) f is continuous at $(0,0)$

(2) The partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at $(0,0)$

(3) f is differentiable at $(0,0)$

(4) f is not differentiable at $(0,0)$

Qus 113: Let $\left\{(X_k,Y_k)\right\}_{k=1}^{\infty}$ be a sequence of independent and identically distributed (i.i.d) random vector with common joint probability density function

$$f(x,y)=\begin{cases}e^{-y} & \text{if } 0<x<y<\infty\\ 0 & \text{otherwise}\end{cases}$$

For $n=1,2,3,\ldots$ let N_n be a random variable denoting the number of elements in the set $\left\{k:k=1,2,\ldots,n:Y_k\ge 2\right\}$

Then, which of the following statements are true?

(1) $\frac{N_n}{3n}$ converges to e^{-2} with probability one

(2) N_n converges to e^{-2} in probability one

(3) $\frac{N_n}{n}$ converges to $3e^{-2}$ in distribution

(4) $\frac{N_n-3ne^{-2}}{\sqrt{3n}}$ converges in distribution to a normal random variable with mean zero and variance $e^{-2}\left(1-3e^{-2}\right)$

Qus 114: Let $M_2(\mathbb{R})$ denote the $\mathbb{R}$-vector space of 2×2 matrices with real entries. Let

$A=\begin{pmatrix}1 & 2\\ 0 & 3\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 0\\ 1 & 5\end{pmatrix}$

Define a linear transformation $T:M_2(\mathbb{R})\to M_2(\mathbb{R})$ be $T(X)=AXB^t$, where B^t denotes the transpose of matrix B. Which of the following statements are true?

(1) $\det(T)=225$

(2) $\det(T)=-225$

(3) $\text{Trace}(T)=16$

(4) $\text{Trace}(T)=-16$

Qus 115: Let $\text{disc}\,\mathbb{D}=\{z\in\mathbb{C}:|z|<1\}$ and f be a holomorphic function on $\mathbb{D}$ such that the function $g(z)=e^{1/z}f(z)$ on $\mathbb{D}\setminus\{0\}$ is bounded. Which of the following statements are true?

(1) $f(0)=0$

(2) $f(z)=0$ for all $z\in\mathbb{D}$

(3) There exists a non-zero constant c such that $f(z)=ce^{-1/z}$ for all $z\in\mathbb{D}\setminus\{0\}$

(4) There exists a non-zero constant c and a positive integer n such that $f(z)=cz^ne^{-1/z}$ for all $z\in\mathbb{D}\setminus\{0\}$

Qus 116: Consider the following design where the columns represent block and the letters represent treatments:

A	C	A	B	A	B	E
B	D	C	D	D	C	F
E	E	E	F	G	G	G

Then, which of the following statements are true?

(1) The design is a bounded incomplete block design

(2) The design is not connected

(3) The design is bindary

(4) The design is symmetric

Qus 117: For a positive integer n and a subset S of the set of positive integers, let $S(n)$ denote the set $\{S\in S\mid s\le n\}$.

Let X be a subset of the set of positive integers such that $\lim_{n\to\infty}\frac{|X(n)|}{n}=1$. Assume that there exist pairwise disjoint subsets $X_1,X_2,\ldots,X_8$ such that $\bigcup_{i=1}^{8}X_i=X$. Which of the following statements are true?

(1) $\lim_{n\to\infty}\frac{|X_i(n)|}{n}$ exists for all $1\le i\le 8$

(2) $\liminf_{n\to\infty}\frac{|X_i(n)|}{n}\ge 0$ for all $1\le i\le 8$

(3) $\limsup_{n\to\infty}\frac{|X_i(n)|}{n}\ge 1/8$ for some $1\le i\le 8$

(4) $\limsup_{n\to\infty}\frac{|X_i(n)|}{n}<1/8$ for all $1\le i\le 8$

Qus 118: Consider

$X=\{u\mid u:[0,1]\to\mathbb{R}\text{ is continuous and } u(0)=0\}$ with the sup norm $\|u\|=\sup_{x\in[0,1]}|u(x)|$.

Let $T(u)=\int_0^1 u(t)\,dt$ and $S\{|T(u)|:u\in X_i\,\|u\|\le 1\}$. Which of the following statements are true?

(1) S is an unbounded subset of $\mathbb{R}$

(2) S is a bounded subset of $\mathbb{R}$ and $\sup(S)=1$

(3) There exists $u\in X$ such that $\|u\|=1$ and $T(u)=1$

(4) S is closed subset of $\mathbb{R}$

Qus 119: The integral equation

$$u(x)=f(x)+\frac{2}{\pi}\int_0^{\pi}\sin(x-t)u(t)\,dt$$

has a unique solution if

(1) $f(x)=\cos x$

(2) $f(x)=\cos 5x$

(3) $f(x) = \sin x$

4) $f(x) = \sin 5x$

Qus 120: Let R be a non-zero ring with unity such that $r^2 = r$ for all $r \in R$. Which of the following statements are true ?

(1) R is never an integral domain
(2) $r = -r$ for all $r \in R$
(3) Every non-zero prime ideal of R is maximal
(4) R must be a commutative ring

CSIR NET EXAM ANSWER KEY (28 FEBRUARY 2025)

PART:- "A"

1. (2) 2. (2) 3. (1) 4. (2) 5. (3) 6. (4) 7. (2)
8. (2) 9. (1) 10. (4) 11. (4) 12. (4) 13. (2) 14. (4)
15. (4) 16. (1) 17. (1) 18. (1) 19. (3) 20. (2)

PART:- "B"

21. (1) 22. (4) 23. (1) 24. (3) 25. (2) 26. (3) 27. (3)
28. (2) 29. (4) 30. (3) 31. (4) 32. (4) 33. (3) 34. (2)
35. (3) 36. (3) 37. (4) 38. (4) 39. (1) 40. (3) 41. (4)
42. (1) 43. (3) 44. (2) 45. (1) 46. (3) 47. (3) 48. (4)
49. (3) 50. (1) 51. (2) 52. (4) 53. (2) 54. (4) 55. (3)
56. (4) 57. (1) 58. (1) 59. (1) 60. (3)

PART:- "C"

61. (1,2) 62. (2,3,4) 63. (1,2,3) 64. (4) 65. (2) 66. (1,2)
67. (1,2,3,4) 68. (2,3) 69. (1,4) 70. (1,4) 71. (1,2) 72. (1,3)
73. (2,3) 74. (1,3) 75. (2,3) 76. (1,2,3,4) 77. (1,3) 78. (2,4)
79. (1,2) 80. (1,2,3,4) 81. (1,2,3) 82. (1,2,3) 83. (1,2,3)
84. (1,2,4) 85. (2) 86. (2,3) 87. (2,3) 88. (1,3,4) 89. (1,2,3)
90. (1,2) 91. (3,4) 92. (4) 93. (3,4) 94. (1,3) 95. (1,3) 96. (1,2,3)
97. (1,2) 98. (1,4) 99. (3,4) 100. (2,4) 101. (1,2,4) 102. (2,3,4)
103. (1,2,3) 104. (2,3) 105. (1,3,4) 106. (2) 107. (1,3) 108. (1)
109. (1,2,3) 110. (1) 111. (3,4) 112. (1,2,4) 113. (1,3,4)
114. (1,3) 115. (1,2) 116. (1,3,4) 117. (2,3) 118. (2)
119. (1,2,3,4) 120. (2,3,4)

CSIR NET SOLUTION

28-Feb-2025

PART "A"

Q 1. Ans (2)

As we know that if pH value is greater than 7 then it is basic and pH value is less than 7 then it is acidic and if pH value is equal to 7 then it is neutral.

It water of pH 8 is diluted 100 times with neutral water of pH 7 then it's pH value will be greater than 7, so it remains basic.

So, option (2) is correct.

Q 2. Ans (2)

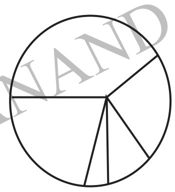
Given TEST = 2182
EAST = 1382
EXAM = 1937

So, $M=7,\ A=3, T=2, E=1$

$\Rightarrow$ MATE = 7321

Hence option (2) is correct.

Q 3. Ans (1)



The above is the representation of five subject by venn diagram.

So, option (1) is correct.

Q 4. Ans (2)

Here in the four digit numbers under the given condition 1 & 2 must be present, so from remaining 3 digits choose any two of them in $3C_2$ ways. Now

Case I:- $\underline{\ 1\ }\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$: $3! = 6$ ways

Case II:- $\underline{\quad}\ \underline{\ 1\ }\ \underline{\quad}\ \underline{\quad}$ $= 2\times 2!$ ways $=4$ ways
$\uparrow$
2 ways

Case III:- $\underline{\quad}\ \underline{\quad}\ \underline{\ 1\ }\ \underline{\quad}$ $= 2!$ ways $=2$ ways

So total number of 4-digit numbers $= 3C_2\left(6+4+2\right) = 3\times 12 = 36$.

So, option (2) is correct.

Q 5. Ans (3)

Total number of handshakes $=19+18+17+16+15+14=99$ as first person hand shaked with remaining 19 and then second person handshaked with remaining 18 and so on 6th person hand shaked with remaining 14 persons.

So, option (3) is correct.

Q 6. Ans (4)

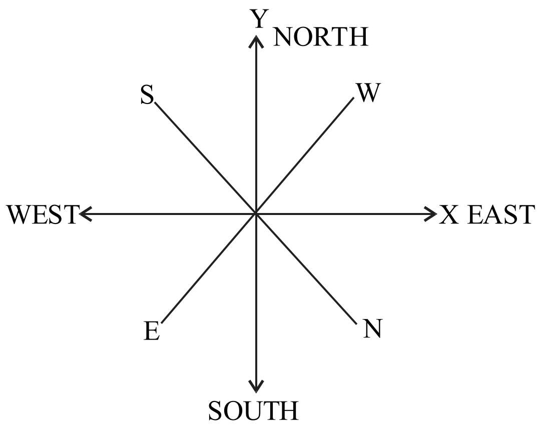
Average of 7 numbers is 71, so their sum is $7\times 71 = 497$.

If x is excluded then average of remaining 6 numbers will be 75 so their sum will be $6\times 75 = 450$,

so the number $x = 497 - 450 = 47$

So, option (4) is correct.

Q 7. Ans (2)



From the above picture if south west becomes east, then north will become southwest.

So, option (2) is correct.

Q 8. Ans (2)

According to the given condition

First $\rightarrow$ TOM

Second $\rightarrow$ Sam (X) David

Third $\rightarrow$ Sam

Fourth $\rightarrow$ Frank

So option (2) is correct.

Q 9. Ans (1)

We know that from two distinct points at most one straight line (in fact exactly one) can be drawn.

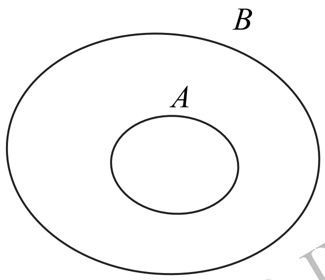
So number of distinct straight lines that can be formed by pairs among 15 points on circle will be at most

$$^{15}C_2 = \frac{15\times 14}{2} = 15\times 7 = 105.$$

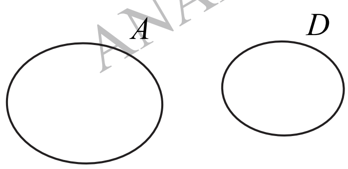
So, option (1) is correct.

Q 10. Ans (4)

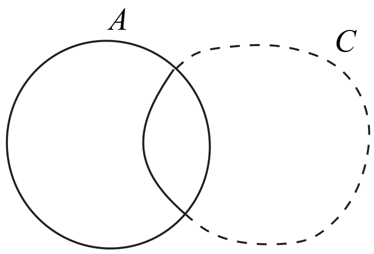
(i) All A are B has Venn diagram as



(ii) No D is A has Venn diagram as



(iii) Some A are C has Venn diagram as



So, (1), (2) & (3) are consistent with it but (4) is not consitent with it.

So, option (4) is correct.

Q 11. Ans (4)

THE NUMBER OF OCCURRENCES OF THE LITTER 'N' IN THIS SENTENCE IS CORRECTLY COUNTED AS

NINE

because if I fill the blanks by SEVEN then number of N will be eight & so on.

So option (4) is correct.

Q 12. Ans (4)

It is given that profit $= 4T - 86$

So, at $25^\circ C$ estimated profit will be

$4\times 25 - 86 = 100 - 86 = 14$

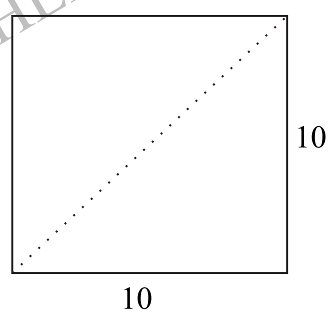
So option (4) is correct.

Q 13. Ans (2)

Among the given four provinces odd one out is CUGHUSTER of fictional country of Numberia.

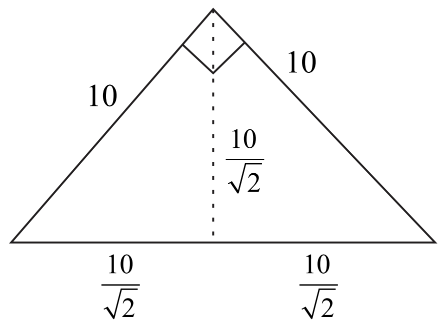
So option (2) is correct.

Q 14. Ans (4)



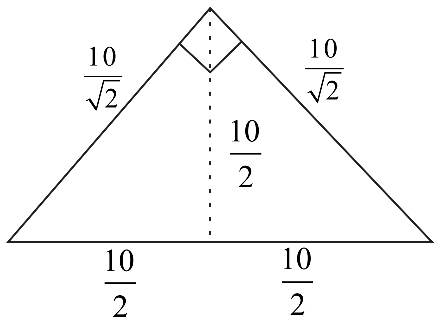
Square Sheet

After folding along diagonal line

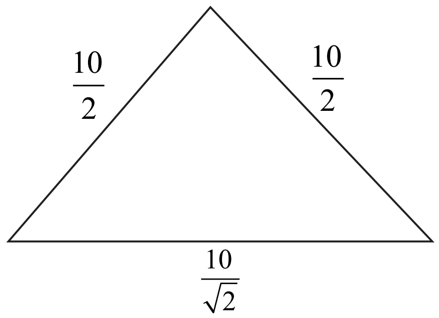


Isosceles right triangle

After first fold it becomes



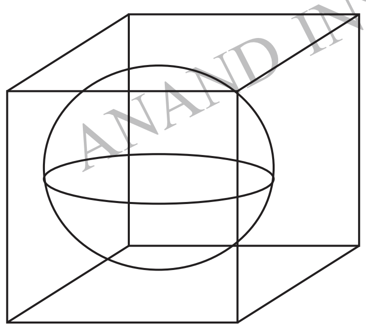
After second fold it becomes



So, length of equal side $=\frac{10}{2}=5$.

So, option (4) is correct.

Q 15. Ans (4)



Let side of the box of length be 'a' then diameter of the ball will be also 'a', so radius of the ball will be $r=\frac{a}{2}$

So, Volume of box $=a^3=V$

& Volume of ball which is spherical of radius

$r=\frac{a}{2}$ will be $V_1=\frac{4}{3}\pi(r)^3=\frac{4}{3}\pi\left(\frac{a}{2}\right)^3=\frac{\pi}{6}a^3$

$$\approx\frac{22}{7\times 6}a^3=\frac{11}{21}a^3$$

So, empty volume

$$=V-V_1=a^3-\frac{11}{21}a^3=a^3\left(1-\frac{11}{21}\right)=\frac{10}{21}a^3$$

So, % of empty volume

$$=\frac{\frac{10}{21}a^3}{a^3}\times 100\%=\frac{1000}{21}\%=47.6\%$$

$\approx 48\%$.

So, option (4) is correct.

Q 16. Ans (1)

In a game we have toss followed by match followed by result and then trophy is given.
So correct chrorological order is
C,A,D,B i.e.
Toss, Match, Result, Trophy.

So, option (1) is correct.

Q 17. Ans (1)

Let cost of the bottle be Rs. x
So, cost of water will be Rs. $x+15$
So, total cost of water bottle
$=x+x+15=20 \Rightarrow 2x=5 \Rightarrow x=2.50$
So, cost of bottle is Rs. 2.50

So, option (1) is correct.

Q 18. Ans (1)

In 2022, no one took admission in the institute then

(i) No one passed the entrance test in 2022
(ii) No one graduated from the institute in 2024
(iii) No one will get job from institute in 2024 but it does not necessarily follows that no one wrote the entrance test in 2024.

So, option (1) is correct.

Q 19. Ans (3)

10 hours $\Rightarrow$ $t=10$ & $r=0.1$
due to exponential growth population after 10 years will be

$$50(1+0.1)^{10}=50(1.1)^{10}=50\times 2.593742601$$

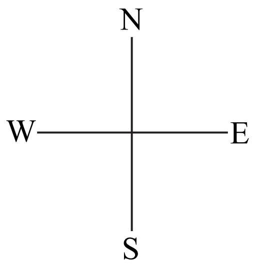
$$=129.6871\approx 130$$

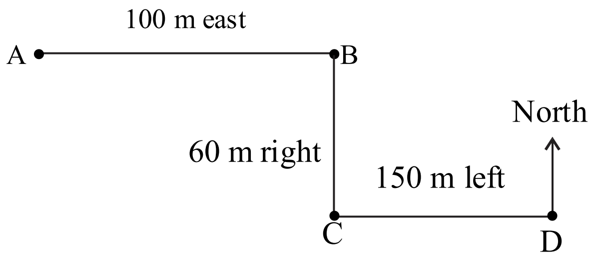
So among the given options correct approximation is 136.

So, option (3) is correct.

Q 20. Ans (2)

Direction is as follows





If I walked east 100 metres, turned right and walled 60 metres, turned left and walked 150 metres and turned left again, I would be facing north from the above representation.

So option (2) is correct.

PART "B"

Q 21. Ans (1) (Statistics)

Q 22. Ans (4)

Iterative formula

$x_{k+1} = g(x_k)$ will converge to the solution of the equation $x = g(x)$

iff $|g'(x)| < 1$ (1)

$\because \quad g(x) = \alpha f(x) + h(x)$

$\Rightarrow \quad g'(x) = \alpha f'(x) + h'(x)$

$\Rightarrow \quad |g'(x)| = |\alpha f'(x) + h'(x)|$ (2)

Given $\sup\limits_{x \neq y} \dfrac{|f(x) - f(y)|}{|x-y|} = L \quad \Rightarrow |f'(y)| \le L$

$\Rightarrow \quad |f'(x)| \le L$ (3)

Also $|h'(x)| \le \dfrac{3}{4}$

From (2), $|g'(x)| = |\alpha f'(x) + h'(x)|$

$\le \alpha |f'(x)| + |h'(x)| \le \alpha L + \dfrac{3}{4} \le 1$

$\Rightarrow \quad \alpha L < 1 - \dfrac{3}{4} = \dfrac{1}{4} \Rightarrow \alpha < \dfrac{1}{4L}$

So option (4) is correct.

Q 23. Ans (1)

$V = \{a_0 + a_1 x + a_2 x^2 \mid a_0, a_1, a_2 \in R\}$

$\Rightarrow \quad V = L(\{1, x, x^2\})$

Now $T(f) = f + \dfrac{df}{dx}$

$\therefore \quad T(1) = 1, T(x) = x + 1 \,\&\, T(x^2) = x^2 + 2x$

so, matrix of linear transformation T is

$$T = \begin{array}{c} \\ 1 \\ x \\ x^2 \end{array} \begin{array}{c} \begin{array}{ccc} T(1) & T(x) & T(x^2) \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

So, eigen values of T are $1,1\,\&\,1$ and characteristic polynomial of T is $C_T(x)=(x-1)^3$ and by Cayley Hamilton theorem

$C_T(T)=0 \Rightarrow (T-I)^3=0 \Rightarrow$

$T^3-3T^2+3T-I=0 \Rightarrow T^3-3T^2+3T=I$

$\Rightarrow \quad \left(T^3-3T^2+3T\right)^{2025}=I$

$\Rightarrow \quad \left(T^3-3T^2+3T\right)^{2025}(x)=I(x)=x$

So, option (1) is correct.

Q 24. Ans (3)

$f(z)=u(x,y)+iv(x,y)$

$\Rightarrow \quad |f(z)|^2=u^2+v^2$

Now $\psi(z)=|f(z)|^2=u^2+v^2$

$\Rightarrow \quad \dfrac{\partial^2\psi}{\partial x^2}=\dfrac{\partial}{\partial x}\left(\dfrac{\partial\psi}{\partial x}\right)=\dfrac{\partial}{\partial x}(2uu_x+2vv_x)$

$=2uu_{xx}+2vv_{xx}+2u_x^2+2v_x^2 \qquad (1)$

similarly

$\dfrac{\partial^2\psi}{\partial y^2}=2uu_{yy}+2vv_{yy}+2u_y^2+2v_y^2 \qquad (2)$

(1) + (2) $\Rightarrow \dfrac{\partial^2\psi}{\partial x^2}+\dfrac{\partial^2\psi}{\partial y^2}=0$

$\Rightarrow \quad 2u(u_{xx}+u_{yy})+2v(v_{xx}+v_{yy})+$

$2(u_x^2+u_y^2+v_x^2+v_y^2)=0$

$\because \quad u\,\&\,v$ are harmonic functions

$\therefore \quad u_{xx}+u_{yy}=0\,\&\,v_{xx}+v_{yy}=0$

$\Rightarrow \quad u_x^2+u_y^2+v_x^2+v_y^2=0$

$\Rightarrow \quad u_x=u_y=v_x=v_y=0$

$\Rightarrow \quad u\,\&\,v$ both are constant, so $f(z)=u+iv$ is constant function. Also, $f(0)=0$, so $f(z)=0$, hence

(1) f can be extendend to C as an entire function.

(2) f must have infinitely many zeros in D

(3) f is 100% a polynomial.

(4) $\exp(f)=e^0=1$, so it cannot take all complex values.

so statement in option (3) is false, so option (3) is correct.

Q 25. Ans (2)

$f(z)=e^{(\cos(1+i))\sin z}$

$\Rightarrow \quad f'(z)=e^{(\cos(1+i))\sin z}.\cos(1+i)\cos z$

$\Rightarrow \quad f'(0)=\cos(1+i)=u_x(0,0)+iv_x(0,0)$

$\Rightarrow \quad u_x(0,0)=\operatorname{Re}(\cos(1+i))$

$=\operatorname{Re}(\cos 1\cosh 1-i\sin 1\sinh 1)$

$=\cos 1\cosh 1=\dfrac{\cos 1}{2}\left(e+\dfrac{1}{e}\right).$

So, option (3) is correct.

Q 26. Ans (2)

We know that for B.V.P. in $[0,l]$ given by $a_0u''+\lambda u=0$ we have

$u=\lambda\int_0^l K(x,t)\,u(t)\,dt$

where, $K(x,t)=\begin{cases} f_1\ ;\ 0\le x\le t \\ f_2\ ;\ t\le x\le l\end{cases}$

satisfying

(i) K is continuous at $x=t$

(ii) K satisfy boundary condition

(iii) $\left(\dfrac{\partial K}{\partial x}\right)_{x\to t^+}-\left(\dfrac{\partial K}{\partial x}\right)_{x\to t^-}=-\dfrac{1}{a_0}$

In our case, $a_0=1\Rightarrow -\dfrac{1}{a_0}=-1$

$l=1, u_x(0)=u(0)$ & $u_x(1)=0$

The value of $K(x,t)$ in option (2) only satisfy all the conditions. So option (2) is correct.

Q 27. Ans (3)

Given $U=L\left(\{e^t,e^{2t},e^{3t}\}\right)$

so $\dim U=3$

Now, $V=\{f:U\to R\,|\,f$ is a an R-linear transformation}

So, $\dim V=\dim(L(U,R))=$

$\dim(U)\dim(R)=3\times 1=3$

Also, $W=\left\{f\in V\,|\,f\left(e^{3t}\right)=0\right\}$

$\Rightarrow\quad \dim W=\dim V-1=3-1=2$,

So, option (3) is correct.

Q 28. Ans (2) (Statistics)

Q 29. Ans (4)

Minimise

$J(y)=\int_0^1\left(y^2+y'^2\right)dx$ subject to

$y(0)=0, y(1)=0$

& $\quad \int_0^1 y^2\,dx=1 \qquad (1)$

Now, $\int_0^1\left(\left(y^2+y'^2\right)+\lambda y^2\right)dx=J'(y) \quad (2)$

So, $f(x,y,y')=y^2(1+\lambda)+y'^2$

$\Rightarrow\quad \frac{\partial F}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$ (E.L.equation)

$\Rightarrow\quad 2y(1+\lambda)-\frac{d}{dx}(2y')=0$

$\Rightarrow\quad 2y(1+\lambda)-2y''=0$

$\Rightarrow\quad y''-(1+\lambda)y=0; y(0)=y(1)=0$

$\Rightarrow\quad y''-\lambda' y=0\;;\;\lambda'=(1+\lambda)$

It is S.L. B.V.P.

For non-trivial solution

$\lambda'<0$ i.e. $\lambda'=-p^2$ & solution will be

$y=C_1\cos px+C_2\sin p(x)$

Now, $y(0)=0\Rightarrow C_1=0$

And, $y(1)=0\Rightarrow C_2\sin p=0\Rightarrow \sin p=0$

$\Rightarrow p=n\pi; n\in I/\{0\}$

$\Rightarrow\quad y=C_n\sin n\pi x$

$\Rightarrow\quad S=\{C_n\sin n\pi x\,|\,n\in I\setminus\{0\}\}$

Now $\int_0^1 y^2dx=1\;\Rightarrow\; C_n^2\int_0^1\sin^2 n\pi x\,dx=1$

$\Rightarrow\quad C_n^2\int_0^1\frac{1-\cos 2n\pi}{2}dx=1$

$\Rightarrow\quad C_n^2\left[\dfrac{x-\dfrac{\sin 2n\pi x}{2n\pi}}{2}\right]_0^1=1$

$\Rightarrow\quad C_n^2\times\frac{1}{2}=1\Rightarrow C_n^2=2\Rightarrow C_n=\pm\sqrt{2}$

$\Rightarrow\quad y=\pm\sqrt{2}\sin n\pi x$

$\Rightarrow\quad \phi(x)=\pm\sqrt{2}\sin n\pi x$

$\Rightarrow\quad \phi\left(\frac{1}{2}\right)=\pm\sqrt{2}\sin\frac{n\pi}{2}$

For $n=1,3,5,\ldots\;\phi\left(\frac{1}{2}\right)=\pm\sqrt{2}$

& for $n=2,4,6,\ldots\qquad \phi\left(\frac{1}{2}\right)=0$

So set $\left\{\phi\left(\frac{1}{2}\right):\phi\in S\right\}=\left\{\sqrt{-2},0,\sqrt{2}\right\}$.

So, option (4) is correct.

Q 30. Ans (3) (Topology)

Q 31. Ans (4)

$B:R^4\times R^4\to R$ is given by

$B(x,y)=x_1y_3+x_2y_4-x_3y_1-x_4y_2\;;$

$x=(x_1,x_2,x_3,x_4)$ & $y=(y_1,y_2,y_3,y_4)$

Now matrix of Bilinear form is $A=\left[a_{ij}\right]_{4\times 4}$ such that

$a_{ij}=B(\alpha_i,\alpha_j);\alpha_i=(0,-0,1,0);i,j=1,2,3,4$

$x \backslash y$	$(1,0,0,0)$	$(0,1,0,0)$	$(0,0,1,0)$	$(0,0,0,1)$
$(1,0,0,0)$	0	0	1	0
$(0,1,0,0)$	0	0	0	1
$(0,0,1,0)$	−1	0	0	0
$(0,0,0,1)$	0	−1	0	0

$$\Rightarrow A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

So, A is real orthogonal and skew symmetric matrix whose eigenvalues are $i, i, -i \,\&\, -i$ &

$\det(A) = (i)^2(-i)^2 = 1$

So option (1) & (2) are wrong.

$B(x,x) = x_1x_3 + x_2x_4 - x_3x_1 - x_4x_2 = 0$;

$\forall x \in R^4$.

So, option (3) is false.

Now if $x \neq 0$ then at least one x_i is non-zero, so corresponding factor of exactly one non-zero x_i take $y_i = 0$ and rest y_i should be taken as 0 then, $\exists y \in R^4$ such that $B(x,y) \neq 0$.

So, option (4) is correct.

Q 32. Ans (4)

For power series

$$\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}} x^n$$

$$a_n = \frac{n^{n^2}}{(n+1)^{n^2}} \Rightarrow |a_n|^{\frac{1}{n}} = \frac{n^n}{(n+1)^n}$$

$$\Rightarrow \lim_{n\to\infty} |a_n|^{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} = \frac{1}{e}$$

So radius of convergence of the power series is $\dfrac{1}{1/e} = e$

So, the power series converges absolutely $\forall x \in R$ s.t. $|x| = e$ & $\forall x \in R$ s.t. $|x| > e$ power series is not convergent.

Hence at $x = 3\,\&$ at $x = 5$ power series is not convergent.

Also series converges $\forall x$ s.t. $|x| < \dfrac{1}{2}$.

so, option (4) is correct only.

Q 33. Ans (3)

Given property

(A) Every non-trivial group homomorphism from G to any group is injective.

Since A_5 is simple group, whose normal sub-groups are given by

$H_1 = \{I_d\}$ and $H_2 = A_5$

Let f be non-trivial group homomorpshim from A_5 to G where G is any group.

i.e. $f : A_5 \to G$ is non-trivial

$\Rightarrow$ $\ker f \neq A_5$

As we know that $\ker f \lhd A_5$

$\Rightarrow$ $\ker f = \{I_d\}$

$\Rightarrow$ f is 1-1

option (3) is correct.

Option (1),(2) and (4) are non-simple groups , hence there exist a non-trivial normal sub-groups.

Since normal subgroup are kernal of some group homomorphism

$\Rightarrow$ There exist a group homomorphim with ker-nel as non-trivial normal subgroups.

$\Rightarrow$ f is not one-one for some group homomor-phism from Z_6 *or* S_3 *or* D_5 to G.

so, options (1), (2) and (4) are incorrect.

so, option (3) is correct only.

Q 34. Ans (2)

Cauchy problem

$xu_x + yu_y = y$; $(x,y) \neq (0,0)$

$\& u(x,1) = \sqrt{1+x^2}$, $x \in R$

has auxiliary equation

$$\frac{dx}{x} = \frac{dy}{y} = \frac{du}{u}; \qquad (1)$$

From (1) & (2) $\dfrac{y}{x} = C_1$ & from 1 & (3) $\dfrac{u}{x} = C_2$

$$\Rightarrow \frac{u}{x} = \phi\left(\frac{y}{x}\right) \Rightarrow u(x,y) = x\phi\left(\frac{y}{x}\right)$$

$$u(x,1)=\sqrt{1+x^2} \Rightarrow x\phi\left(\frac{1}{x}\right)=x\sqrt{1+\left(\frac{1}{x}\right)^2}$$

$$\Rightarrow \quad \phi\left(\frac{1}{x}\right)=\sqrt{1+\left(\frac{1}{x}\right)^2} \Rightarrow \phi\left(\frac{y}{x}\right)=\sqrt{1+\left(\frac{y}{x}\right)^2}$$

$$\Rightarrow \quad u(x,y)=x\sqrt{1+\left(\frac{y}{x}\right)^2}=\sqrt{x^2+y^2}$$

As, $u(1,0)=1, u(1,y)=\sqrt{1+y^2}$, so (1) & (3) are false.

$$u(x_1,y_1)=u(x_2,y_2) \Rightarrow \sqrt{x_1^2+y_1^2}=\sqrt{x_2+y^2}$$

$\Rightarrow \quad x_1^2+y_1^2=x_2^2+y_2^2$. So (2) is true & (4) is false.
So, option (2) is correct.

Q 35. Ans (3) (Statistics)

Q 36. Ans (3) (Statistics)

Q 37. Ans (4) (Statistics)

Q 38. Ans (4)

$$\lim_{n\to\infty}\int_0^1\int_0^1\ldots\int_0^1 \frac{x_1^2+x_2^2+\ldots+x_n^2}{x_1+x_2+\ldots+x_n}dx_1,\ldots,dx_n$$

$$=\frac{\int_0^1 x^2\,dx}{\int_0^1 x\,dx} \quad \text{(By central limit theorem.)}$$

$$=\frac{1/3}{1/2}=\frac{2}{3}.$$

So, option (4) is correct.

Q 39. Ans (1)

$y_1(x)=e^{2x}$ is a solution of ODE

$$x\frac{d^2y}{dx^2}-(3+4x)\frac{dy}{dx}+(4x+6)y=0 \qquad (1)$$

Now we know that if $y_1(x)$ is a solution of $a_0y''+Py'+Qy=0$ then it's second solution $y_2(x)$ is

$$y_2=y_1\int\frac{1}{y_1^2}\,e^{-\int\frac{P}{a_0}dx}$$

$$\Rightarrow \quad y_2=e^{2x}\int\frac{1}{e^{4x}}e^{-\int\frac{-(3+4x)}{x}dx}dx$$

$$=e^{2x}\int\frac{1}{e^{4x}}e^{3\ln x+4x}\,dx$$

$$\Rightarrow \quad y_2(x)=e^{2x}\int\frac{e^{4x}.x^3}{e^{4x}}dx=e^{2x}.\frac{x^4}{4}$$

$$\Rightarrow \quad y_2(x)=\frac{e^{2x}x^4}{4}$$

$$\Rightarrow \quad y_2'(x)=\frac{2e^{2x}x^4+4x^3e^{2x}}{4}$$

$$=\frac{2e^{2x}x^3(x+2)}{4}=\frac{e^{2x}x^3(x+2)}{2}$$

```
     +        -        +
 ---------|--------|---------
         -2        0
```

sign scheme of $y_2'(x)$

So, $y_2(x)$ is strictly increasing in $(0,\infty)$ so option (1) is correct and option (3) is incorrect . Also, $e^{-2x}y_2(x)=\frac{x^4}{4}\to\infty$ as $x\to\infty$
So, option (2) & (4) are incorrect.

Q 40. Ans (3)

Q 41. Ans (4)

Solution of given wave equation $u_{tt}-u_{xx}=0; x\in R; t>0$ satisfying the given condition $u(0,t)=0;\ \forall t\geq 0$ will be particularly of the form $u(x,t)=x$ (1)

So, from (1) $u(t,t)=t>0$

$\because t>0$ is given , so option (1) is false.

For $x=-t$, $u(x,t)=-t<0$
So option (2) is false.

$u(-x,t)=-x\neq x=u(x,t)$
So, option (3) is false

$u(-x,t)=-x=-u(x,t); 0<x\leq t$
So, option (4) is true only.

Q 42. Ans (1)

H is upper half plane & $f: H \to C$ is non constant holomorphic function such that $|f(z)| < 1$ for all $z \in H$, so

$$f(z) = a\,e^{i\phi}\frac{z - k\,i}{z + k\,i}; |a| < 1, k \in R^{+}, \phi \in R$$

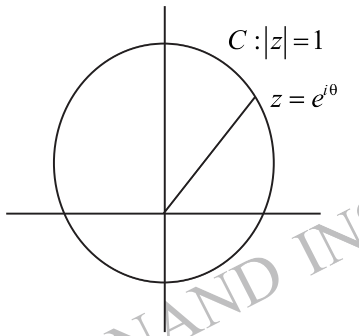
$$\Rightarrow \quad f'(z) = a\,e^{i\phi}\frac{(z + k\,i) - (z - i\,k)}{(z + k\,i)^2} = \frac{a\,e^{i\phi}\,2k\,i}{(z + k\,i)^2}$$

$$\Rightarrow \quad f'(i\,y) = \frac{a\,e^{i\phi}\,2k\,i}{i^2 (y + k)^2} \Rightarrow \lim_{y \to \infty} f'(i\,y) = 0$$

Q 43. Ans (3) (Statistics)

Q 44. Ans (2)

$m, n \in N$ and $I_{m,n} = \frac{1}{2\pi i}\int_{C:|z|=1} z^m \bar{z}^n \; dz$



On $C: z = e^{i\theta} \Rightarrow dz = i\,e^{i\theta}\,d\theta$ & $\bar{z} = e^{-i\theta}$

So, $I_{m,n} = \frac{1}{2\pi i}\int_{\theta=0}^{2\pi} i\,e^{i(m+1-n)\theta} d\theta$

$$= \frac{1}{2\pi i}\int_{\theta=0}^{2\pi} e^{i(m+1-n)\theta} d\theta$$

$= 0$ if $m + 1 - n \neq 0$

$= 1$ if $m + 1 - n = 0$

So, $I_{m,n} = 1$ if $m + 1 - n = 0$ i.e. $m + 1 = n$.

Q 45. Ans (1)

Q 46. Ans (3)

$$f_n(x) = \frac{x}{(1-x)^n}; x \in [-1, 0]$$

$$\Rightarrow \quad \sum_{n=0}^{\infty} f_n(x) = S(x) = 0; x = 0$$

$$= x\left(1 + \frac{1}{(1-x)} + \frac{1}{(1-x)^2} + \ldots\right)$$

$$= \frac{x}{1 - \frac{1}{1-x}} = \frac{x}{\frac{1-x-1}{1-x}} = x - 1 \;;\; x \in [-1, 0)$$

$\because \quad \sum_{n=0}^{\infty} f_n(x)$ is continuous in $[-1, 0]$ but it's sum function $S(x)$ is not continuous in $[-1, 0]$, so convergence is not uniform.

Now $\sum_{n=0}^{\infty} |f_n(x)| = \sum_{n=0}^{\infty}\left|\frac{x}{(1-x)^n}\right|$

$$= \sum_{n=0}^{\infty}\frac{|x|}{(1-x)^n}$$

$\because x \in [-1, 0]$

$\Rightarrow 1 - x \in [1, 2]$

so $(1 - x) > 0$ in $[-1, 0]$

Hence $\sum_{n=0}^{\infty} |f_n(x)| = 0 \;;\; x = 0$

$$= \frac{|x|}{1 - \frac{1}{1-x}} = \frac{|x|(1-x)}{-x} = (1 - x); \; x \in [-1, 0)$$

$\because \quad |x| = -x \;;\; \forall\, x \in [-1, 0)$

So $\sum_{n=0}^{\infty} f_n(x)$ is absolutely convergent.

Hence option (3) is correct.

Q 47. Ans (3)

Recall the result.

Let R be C.R.U. then $\frac{R}{I}$ is I.D. iff I is prime ideal $\frac{\mathbb{C}[x, y]}{\langle x, y\rangle} \approx \mathbb{C} \Rightarrow \frac{\mathbb{C}[x, y]}{\langle x, y\rangle}$ is an Integral domain.

$$\frac{\mathbb{C}[x,y]}{\langle x+y\rangle}\approx\mathbb{C}\Rightarrow\frac{\mathbb{C}[x,y]}{\langle x+y\rangle}$$ is an I.D.

The ideal $I=\langle xy-1\rangle$ is generated by an irreducible polynomial in $\mathbb{C}[x,y]$

$\Rightarrow$ $\frac{\mathbb{C}[x,y]}{\langle xy-1\rangle}$ is field

$\Rightarrow$ $\frac{\mathbb{C}[x,y]}{\langle xy-1\rangle}$ is an I.D.

So,options (1), (2) and (4) are false.
option (3) is correct ,

Since the ideal $\langle x^2+y^2\rangle$ is not prime ideal.

As $(x+iy)(x-iy)\in\langle x^2+y^2\rangle$ but

$(x+iy),(x-iy)\notin\langle x^2+y^2\rangle$

$\Rightarrow$ $\frac{\mathbb{C}[x,y]}{\langle x^2+y^2\rangle}$ is not I.D.

option (3) is correct ,

Q 48. Ans (4)

Q 49. Ans (3) (Statistics)

Q 50. Ans (1)

$$a_n=\frac{e^n+e^{-n}}{2}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{e^n+e^{-n}}{2}$$

$$=\frac{\infty+0}{2}=\infty$$

So, sequence $\{a_n\}$ diverges to ∞, hence for each $x\in R, \exists n\in N$ s.t. $a_m>x$; $\forall m\geq n$

so, $\exists n\in N$ s.t. $a_n>x$. So, option (1) is correct and option (2) is incorrect.

Also $b_n=\frac{a_{n+1}}{a_n}=\frac{e^{n+1}+e^{-(n+1)}}{e^n+e^{-n}}$

$$=\frac{e^{n+1}\left(1+e^{-2(n+1)}\right)}{e^n\left(1+e^{-2n}\right)}=\frac{e\left(1+e^{-2(n+1)}\right)}{1+e^{-2n}}$$

$$\Rightarrow\quad\lim_{n\to\infty}b_n=\lim_{n\to\infty}\frac{e\left(1+e^{-2(n+1)}\right)}{\left(1+e^{-2n}\right)}$$

$=e$

So, $\{b_n\}$ converges to e and all terms of $\{b_n\}$ are positive so for $x=100000\in R$, there is no n s.t. $a_n>x$.
So option (3) is false.
and also for $x=-1$, there is no n s.t. $a_n<x$
So, option (4) is false.

Hence option (1) is correct only.

Q 51. Ans (2)

Row space of the matrix $A=\begin{pmatrix}2&2&7\\3&1&4\end{pmatrix}$

over field R, is subspace of R^3 generated by vectors $(2,2,7)$ & $(3,1,4)$,

i.e. Rowspace $(A)=L(\{(2,2,7),(3,1,4)\})$

Now if $u=(a,b,c)\in R^3$ is non-zero vector lying in orthogonal complement of Rowspace(A) then

$\langle(a,b,c),(2,2,7)\rangle=0$ &

$\langle(a,b,c),(3,1,4)\rangle=0$

$$\Rightarrow\quad\begin{aligned}2a+2b+7c&=0\\3a+b+4c&=0\end{aligned}$$

$\Rightarrow$ $\frac{a}{1}=\frac{b}{13}=\frac{c}{-4}=K$ (say) then

$a=K, b=13K$ & $c=-4K$
So,
$|a+b+c|=|K+13K-4K|=|10K|=10|K|$

Now, a,b,c are integers, and (a,b,c) is non-zero vector, so minimum value of $|a+b+c|$

$=\min_{K\in I\setminus\{0\}}(10|K|)=10(|-1|)=10$.

So option (2) is correct.

Q 52. Ans (4) (Statistics)

Q 53. Ans (2)

Q 54. Ans (4)

$$A=\begin{bmatrix}0 & a & 0\\ 0 & 0 & b\\ c & 0 & 0\end{bmatrix};\overset{a}{(1,2)},\ \overset{b}{(2,3)},\ \overset{c}{(3,1)}$$

$$\Rightarrow\quad A^2=\begin{bmatrix}0 & 0 & ab\\ bc & 0 & 0\\ 0 & ca & 0\end{bmatrix};\ \overset{ab}{(1,3)},\overset{bc}{(2,1)},\overset{ca}{(3,2)}$$

$$\Rightarrow\quad A^3=\begin{bmatrix}abc & 0 & 0\\ 0 & abc & 0\\ 0 & 0 & abc\end{bmatrix}=I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\because\quad abc=1$

$\Rightarrow\quad B=A+A^2+A^3$

$$=\begin{bmatrix}abc & a & ab\\ bc & abc & b\\ c & ca & abc\end{bmatrix}$$

Now by elementary row transformations $R_2\to R_2-aR_1$ & $R_3\to R_3-cR_1$ and using $abc=1$ we get

$$B\sim\begin{bmatrix}abc & a & ab\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

So, Rank $(B)=1$

and hence $\det(B)=0$

So, options (1) & (3) are false

Also $A^3=I\Rightarrow A^3$ is non-singular matrix, so

$|A^3|\neq 0\Rightarrow |A^3|\neq 0$

$\Rightarrow\quad |A|\neq 0$

So, option (2) is false.

Also Rank $(B)=1\Rightarrow$ Rank $(B^2)\leq 1$ (1)

& $B=\begin{bmatrix}1 & a & ab\\ bc & 1 & b\\ c & ca & 1\end{bmatrix}$

$\Rightarrow\quad (1,1)^{th}$ element of $B^2=1^2+abc+abc$

$=3\neq 0$

So, $B^2\neq 0$

$\Rightarrow\quad$ Rank $(B^2)\geq 1$ (2)

(1) & (2) $\Rightarrow$ Rank $(B^2)=1$

Hence option (4) is correct only.

Q 55. Ans (3)

$$A=\lim_{n\to\infty}\left(\sum_{K=1}^{n}f\left(\frac{K}{n}\right)-n\int_0^1 f(x)\,dx\right)$$

$$=\lim_{n\to\infty}n\left(\frac{1}{n}\sum_{K=1}^{n}f\left(\frac{K}{n}\right)-\int_0^1 f(x)\,dx\right)$$

$$=\lim_{n\to\infty}n\left[\begin{array}{l}\frac{1}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+\ldots+f\left(\frac{n}{n}\right)\right)\\ -\left\{\int_0^{1/n}\frac{f(0)+f\left(\frac{1}{n}\right)}{2}dx+\int_{1/n}^{2/n}\frac{f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)}{2}dx+\ldots\int_{n-1/n}^{n/n}\frac{f\left(\frac{n-1}{n}\right)+f\left(\frac{n}{n}\right)}{2}dx\right\}\end{array}\right]$$

$$=\lim_{n\to\infty}n\left[\begin{array}{l}\frac{1}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+\ldots+f\left(\frac{n}{n}\right)\right)\\ -\frac{1}{n}\left\{\frac{f(0)+2\left(f\left(\frac{1}{n}\right)+\ldots+f\left(\frac{n-1}{n}\right)\right)+f\left(\frac{n}{n}\right)}{2}\right\}\end{array}\right]$$

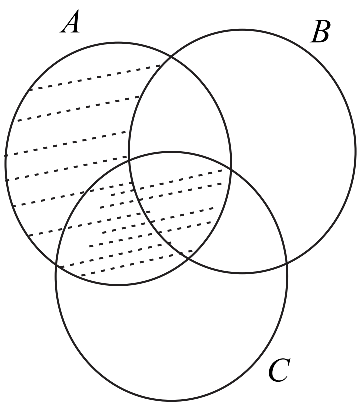
$$=\lim_{n\to\infty}\frac{1}{n}.n\left[\frac{f\left(\frac{n}{n}\right)-f(0)}{2}\right]$$

$$= \frac{f(1) - f(0)}{2} = \frac{\sin(1^2) - \sin(0^2)}{2}$$

$$= \frac{\sin(1)}{2} \quad \because f(x) = \sin x^2.$$

Hence option (3) is correct only.

Q 56. Ans (4)



From the above figure

$A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$.

Hence option (4) is correct only.

Q 57. Ans (1)

Q 58. Ans (1)

Given $V = M_5(R)$

& $S = \{AB - BA \mid A, B \in V\}$

$W = L(S)$

Also $T : V \to R$ is linear transformation such that $T(A) = tr(A)$

So, kernel of T or null space of T is

$$\ker(T) = \{A \in V \mid tr(A) = 0\} \qquad (1)$$

Also $tr(AB - BA) = tr(AB) - tr(BA)$

$$= 0 \quad \because tr(AB) = tr(BA)$$

So, S is collection of all matrices from V whose trace is 0.

Hence, $W = \ker(T)$.

So, option (1) is correct.

Q 59. Ans (1)

Given D.E. is

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + e^{2x}y = 0, x \in R \qquad (1)$$

to convert it into D.E. with constant coefficient with independent variable $S = S(x)$ & $\frac{dS}{dx}(0) = 1$ we take $S = e^x$ so that

$$\frac{dS}{dx}(0) = e^x\Big|_{x=0} = 1$$

So, $\frac{dy}{dx} = \frac{dy}{ds} \cdot \frac{ds}{dx} = e^x \frac{dy}{ds} = s\frac{dy}{ds}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{ds}\left(\frac{dy}{dx}\right) \cdot \frac{ds}{dx}$$

$$= \frac{d}{ds}\left(s\frac{dy}{ds}\right) \cdot s = s^2\left(\frac{d^2y}{ds^2}\right) + s\frac{dy}{ds}$$

So, (1) becomes

$$s^2\frac{d^2y}{ds^2} + s\frac{dy}{ds} + p(x) \cdot s\frac{dy}{ds} + s^2y = 0$$

$$\Rightarrow \quad s^2\frac{d^2y}{ds^2} + s(1 + p(x))\frac{dy}{ds} + s^2y = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \left(\frac{1 + p(x)}{s}\right)\frac{dy}{ds} + y = 0$$

Which will be D.E. with constant coefficient of $\frac{1 + p(x)}{s}$ is constant

$\Rightarrow \quad \frac{1 + p(x)}{e^x}$ is constant

$\Rightarrow \quad e^{-x}(1 + p(x))$ is constant

So option (1) is correct.

Q 60. Ans (3)

It is well known results in number theory.

If $\gcd(n,\phi(n))>1$, then there exist more than one up to isomorphic or non-isomorphic groups.

$\Rightarrow$ Option (3) is correct.

(1) is not correct since $G(15)=1$ but 15 is not prime.

(2) $G(8)=5$, i.e. if $O(G)=8$, then there exist exactly 5-non isomorphic group of order 8

$\Rightarrow$ $G(8)=5$

Option (2) is not correct.

(4) From the justification for option (2), we conclude that option (4) is not correct.

OR

Also we can say that if $O(G)$ is sufficierly large then number of non-abelian groups exponentially increasing.

Hence option (3) is correct only.

PART "C"

Q 61. Ans (3) (Statistics)

Q 62. Ans (2) (3) (4)

$f(x)$ inter polates the data $(-1,2),(0,1)$ and $(1,2)$, so by the Forward difference table ,

x	y	Δ	Δ^2
-1	2		
		-1	
0	1		2
		1	
1	2		

and step length is $h=1$, by Newton's Forward difference formula ,

$$f(x)=2+(-1)p+2\frac{p(p-1)}{2!}$$

where $x=-1+p(1)\Rightarrow p=x+1$

$$\Rightarrow f(x)=2-(x+1)+x(x+1)$$

$$\Rightarrow f(x)=x^2+1 \qquad (1)$$

Also $f(x)+g(x)$ is interpolated by $(-1,2),(0,1),(1,2)$ and $(2,17)$ so it's forward difference table will be as follows

x	y	Δ	Δ^2	Δ^3
-1	2			
		-1		
0	1		2	
		1		12
1	2		14	
		15		
2	17			

$$\Rightarrow y(x)=y_0+p\,\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_0+\frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

where $x=x_0+ph\,;x_0=-1\,\&\,h=1$

$\Rightarrow \quad x = -1 + p.1 \Rightarrow p = x + 1$.

$\Rightarrow \quad f(x) + g(x) = 2 - (x+1) + x(x+1)$

$+2x(x^2 - 1)$ (2)

from (1) & (2) (infact (2) - (1))

$g(x) = 2x(x^2 - 1)$ (3)

Now $f(5) = 5^2 + 1 = 26$; $f(1) = 1^2 + 1 = 2$ &

$g(3) = 6(3^2 - 1) = 48$.

So,

(1) $f(5) + g(3) = 26 + 48 = 74$

So option (1) is incorrect.

(2) $2f(5) - g(3) = 2 \times 26 - 48 = 4$

So, option (2) is correct.

(3) $f(1) + g(3) = 2 + 48 = 50$

So, option (3) is correct.

(4) $f(5) + g(3) = 26 + 48 = 74$

So, option (4) is correct.

Hence, options 2,3 & 4 are correct only.

Q 63. Ans (1) (2) (3)

$f : C \to C$ is an entire function, so by Mclaurin's expansion

$f(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 + \dots$ (1)

$\Rightarrow \quad f(iz) = a_0 + i a_1 z - a_2 z^2 - i a_3 z^3 + a_4 z^4 + \dots$ (2)

(1) & (2)

$\Rightarrow \quad a_0 = a_0 ; a_1 = i a_1, a_2 = -a_2, a_3 = -i a_3, a_4 = a_4 \dots$

$\Rightarrow \quad a_1 = a_2 = a_3 = a_5 = a_6 = a_7 = \dots = 0$

& $a_0, a_4, a_8 \dots$ can be any complex value.

Hence, $f(z) = a_0 + a_4 z^4 + a_8 z^8 + \dots$ (3)

So, $f(-z) = a_0 + a_4 z^4 + a_8 z^8 + \dots = f(z)$

So, $f(z) = f(-z)$, option (1) is correct.

Also, from (3), $f'(0) = f''(0) = f'''(0) = 0$

option (2) is correct.

Also, if we take

$g(z) = a_0 + a_4 z^4 + a_8 z^8$ then it is entire functions and $f(z) = g(z^4)$ for all $z \in c$

So, option (3) is correct.

As $f(z) = z^4$ also satisfy the conditions so, f is not necessarily constant function.

So, option (4) is incorrect.

Hence, options 1,2 & 3 are correct only.

Q 64. Ans (4)

Given $M = AB - BA$

Let $A = \begin{bmatrix} a & c \\ 0 & b \end{bmatrix}$ & $B = \begin{bmatrix} d & e \\ 0 & f \end{bmatrix}$

$\Rightarrow \quad AB = \begin{bmatrix} ad & ae + cf \\ 0 & bf \end{bmatrix}$

& $BA = \begin{bmatrix} ad & cd + be \\ 0 & bf \end{bmatrix}$

$\Rightarrow \quad M = AB - BA = \begin{bmatrix} 0 & (a-b)e + c(f-d) \\ 0 & 0 \end{bmatrix}$

So, if a, b, d & f are distinct, then A & B are diagonalisable over R but M need not be diagonalisable as A.M. of e.v. of M will be 2 and it's G.M. can be 1.

Also M as seen from above need not be scalar matrix.

$\because \quad M = AB - BA$

$\therefore \quad tr(M) = tr(AB - BA)$

$= tr(AB) - tr(BA) = 0$

So if eigenvalues of M are x & y then

$x + y = 0 \quad \Rightarrow \quad y = -x$

So, characteristic polynomial of M will be

$c(t) = (t - x)(t + x) = t^2 - x^2$

By Cayley Hamilton theorem.

$C(M) = 0 \Rightarrow M^2 - x^2 I = 0 \Rightarrow M^2 = x^2 I$

$\Rightarrow \quad \exists \lambda \in R$ s.t. $M^2 = \lambda I$

So, option (4) is correct.

Q 65. Ans (2) (Statistics)

Q 66. Ans (1) (2) (Statistics)

Q 67. Ans (1) (2) (3) (4)

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} ; & x \neq 0 \\ 0 ; & x = 0 \end{cases}$$

$$\Rightarrow \quad \lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 \sin\frac{1}{x} = 0.$$

So it exist. so option (1) is correct.

Also, $\lim_{x\to 0} f(x) = 0 = f(0)$, so f is continuous at 0.
So, option (2) is correct.

Now $f'(0) = \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0}$

$$= \lim_{x\to 0} \frac{x^2 \sin\frac{1}{x} - 0}{x} = \lim_{x\to 0} x \sin\frac{1}{x} = 0$$

so f is differentiable at 0.
Further

$$f'(x) = \begin{cases} 2x\sin\frac{1}{x} - \cos\frac{1}{x} \; ; & x \neq 0 \\ 0 \; ; & x = 0 \end{cases}$$

Hence, options 1,2,3 & 4 are correct

Q 68. Ans (2) (3)

For $b \in R$, $y_b = y_b(x)$ is the unique solution of I.V.P.

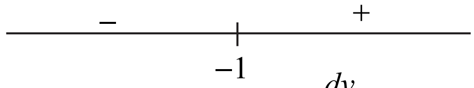
$\frac{dy}{dx} = y^5 + y^4 + y^3 + y^2 + y + 1$, $y(0) = b$ defined on it's maximal interval of existence I_b; then

$\because \quad \frac{dy}{dx} = (y^4 + y^2 + 1)(y + 1)$

At critical points $\frac{dy}{dx} = 0$ i.e.

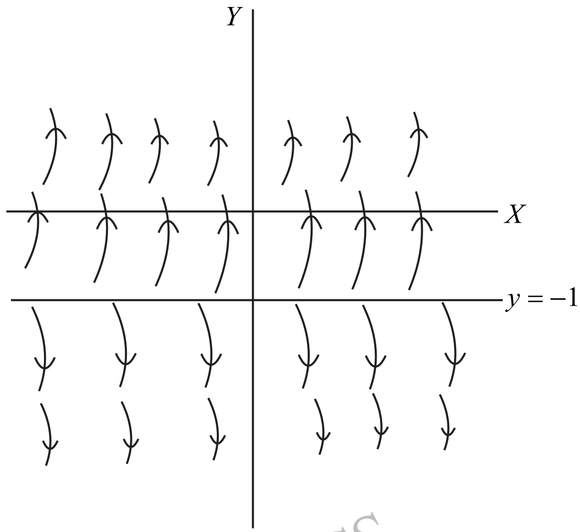
$(y+1)(y^4 + y^2 + 1) = 0 \Rightarrow y + 1 = 0 \Rightarrow y = -1$

$\because \quad y^4 + y^2 + 1 = 0$ has no real roots.



sign scheme of $\frac{dy}{dx}$

So, y is S.I. in $[-1, \infty)$ & S.D. in $(-\infty, -1]$



Hence the solution is bounded below in $[-1, \infty)$ so also in $(0, \infty)$ & solution is bounded above in $(-\infty, -1]$ so in $(-\infty, 0)$.
Hence option 2 & 3 are correct only.

Q 69. Ans (1) (4)

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = \sin\left(e^{-5x}\right), x > 0$$

$$\Rightarrow \quad \left(D^2 + 5D + 6\right) y = \sin\left(e^{-5x}\right)$$

It's auxiliary equation is $m^2 + 5m + 6 = 0$

$\Rightarrow (m+2)(m+3) = 0 \Rightarrow m = -2, -3$ so it's complementary function is

$y = C_1 e^{-2x} + C_2 e^{-3x}$ (1)

It's particular intergral is

$$y = \frac{1}{(D+2)(D+3)} \sin\left(e^{-5x}\right)$$

$$= \frac{(D+3) - (D+2)}{(D+2)(D+3)} \sin\left(e^{-5x}\right)$$

$$= \frac{1}{(D+2)} \sin\left(e^{-5x}\right) - \frac{1}{D+3} \sin\left(e^{-5x}\right)$$

$$= e^{-2x} \int e^{2x} \sin\left(e^{-5x}\right) dx - e^{-3x}$$

$\int e^{3x}\sin\left(e^{-5x}\right)dx$ (2)

So, it's general solution is

$y = C_1e^{-2x} + C_2e^{-3x} + e^{-2x}\int e^{2x}\sin\left(e^{-5x}\right)dx$

$+e^{-3x}\int e^{3x}\sin\left(e^{-5x}\right)dx$ (3)

Now as $x \to \infty, e^{-5x} \to 0$

So, $\sin\left(e^{-5x}\right) \to 0$

As $x \to \infty\, C_1e^{-2x}, C_2e^{-3x}, e^{-2x}\int e^{2x}\sin\left(e^{-5x}\right)dx$

& $e^{-3x}\int e^{3x}\sin\left(e^{-5x}\right)dx$ all tends to 0.

So, $y \to 0$ i.e. solution of ode tends to 0. So option (4) is correct.

Also in $(0,\infty); C_1e^{-2x}, C_2e^{-3x}$ are bounded and

$\int_0^\infty e^{2x}\sin\left(e^{-5x}\right)dx$ & $\int_0^\infty e^{3x}\sin e^{-5x}dx$ are inproper integral which are convergent, so they are bounded so option (1) is correct.

Hence options 1 & 4 are correct only.

Q 70. Ans (1) (4) (Statistics)

Q 71. Ans (1) (2)

Given quadrature formula is

$\int_{x_0}^{x_0+h} f(x)dx \approx \lambda h\left(f(x_0) + f(x_0+h)\right) +$

$ph^3\left(f''(x_0) + f''(x+h)\right)$ (1)

For $f(x) = 1$ equation (1) becomes

$\int_{x_0}^{x_0+h} 1\, dx = \lambda h(1+1) + ph^3(0+0)$

$\Rightarrow \quad h = 2\lambda h \Rightarrow 2\lambda = 1 \Rightarrow \lambda = 1/2$ (2)

For $f(x) = x$ equation (1) becomes

$\int_{x_0}^{x_0+h} x\,dx = \lambda h(2x_0 + h) + ph^3(0+0)$

$\Rightarrow \quad \dfrac{(x_0+h)^2 - x_0^2}{2} = x_0 h + \dfrac{1}{2}h^2 \because \lambda = \dfrac{1}{2}$

which is identity.

For $f(x) = x^2$, equation (1) becomes

$\int_{x_0}^{x_0+h} x^2\,dx = \lambda h\left(x_0^2 + (x_0+h)^2\right) + ph^3(2+2)$

$\Rightarrow \quad \dfrac{(x_0+h)^3 - x_0^3}{3} = \dfrac{h}{2}\left(2x_0^2 + 2hx_0 + h^2\right) + 4ph^3$

$\Rightarrow \quad \dfrac{h\left((x_0+h)^2 + x_0^2 + x_0(x_0+h)\right)}{3} =$

$hx_0^2 + h^2x_0 + h^3\left(\dfrac{1}{2} + 4p\right)$ (3)

From (3) by equating coefficient of h^3, we get

$\dfrac{1}{3} = \dfrac{1}{2} + 4p \Rightarrow 4p = \dfrac{-1}{2} + \dfrac{1}{3} = \dfrac{-1}{6}$

$\Rightarrow \quad p = -\dfrac{1}{24}$ (4)

From (2) & (4)

(1) $2\lambda + 24p = 2\times\dfrac{1}{2} + 24\times\left(-\dfrac{1}{24}\right) = 1 - 1 = 0$

so option (1) is correct.

(2) $7\lambda - 12p = 7\times\dfrac{1}{2} - 12\times\left(-\dfrac{1}{24}\right)$

$= \dfrac{7}{2} + \dfrac{1}{4} = 4$.

so option (2) is correct.

(3) $2\lambda + 24p = 2\times\dfrac{1}{2} + 24\times\left(-\dfrac{1}{24}\right) = 0$

so option (3) is incorrect.

(4) $7\lambda - 12p = 7\times\dfrac{1}{2} - 12\left(-\dfrac{1}{24}\right) = 4$

so option (4) is incorrect

So options (1) & (2) are correct only.

Q 72. Ans (1) (3)

Given vector space is $C[0,\pi](R)$

Now $\sin x$ is non-zero vector from $C[0,\pi]$, $\cos x$ cannot be spanned by $\sin x$, $\sin^2 x$ cannot be spanned by $\sin x \,\&\, \cos x$, & $\cos^2 x$ cannot be spanned by $\sin x$, $\cos x \,\&\, \sin^2 x$

so set $S=\{\sin x,\cos x,\sin^2 x,\cos^2 x\}$ is linearly independent set of vectors. Also $W=L(S)$, so S is a basis of W.

(option 1)

$\|\sin x\|=\sqrt{\langle \sin x,\sin x\rangle}$

$=\sqrt{\int_0^{\pi}\sin^2 x dx}=\sqrt{2\int_0^{\pi/2}\sin^2 x\,dx}$

$=\sqrt{2.\frac{1}{2}.\frac{\pi}{2}}=\sqrt{\frac{\pi}{2}}$ By Walhi's theorem

so $\sin x$ is not unit vector.
Hence S is not orthonormal basis.
(option 2 is incorrect)

Now if we take $f(x)=\sin x$ & $g(x)=\cos x$

then $\langle f,g\rangle=\langle \sin x,\cos x\rangle=$

$\int_0^{\pi}\sin x\cos x\ dx=\int_0^{\pi}\frac{\sin 2x}{2}dx=\left.\frac{\cos 2x}{2\times 2}\right|_0^{\pi}$

$=\frac{1}{4}(1-1)=0$.

So, option (3) is correct.

As $\dim(W)=4$, so S do not contain an orthonormal basis, as all vectors of S are not unit vector, so option (4) is false.
So option (4) is false.
Hence options 1 & 3 are correct only.

Q 73. Ans (2) (3) (Statistics)

Q 74. Ans (1) (3)

Q 75. Ans (2) (5)

Given vector space is $V(Q)$ where $V=L(\{1,x,x^2,x^3\})$ and $\dim V=4$

$A=\{f:V\to Q\mid f \text{ is a Q-linear transformation}\}$

$=L(V(Q),Q(Q))=Hom(V(Q),Q(Q))$

$\Rightarrow\quad \dim A=\dim(L(V(Q),Q(Q)))$

$=\dim V(Q).\dim Q(Q)$

$=4\times 1=4$

So, option (3) is correct.

Also, $B=\{f\in A\mid f(1)=0\}$

i.e. set of all linear function on V which vanishes at 1.

so, $\dim(B)=\dim(A)-1$

$=4-1=3$

So, option (2) is correct.

As $\hat{O}\in A$ & hence if we take $f=\hat{O}$ then for this $f\in A$, image of A will be zero dimensional subspace of Q-vector space.
So, option (4) is incorrect.

Also $\hat{O}\in B$ and for this $f\in B$

$\dim\ker f=4-0=4$

So option (1) is incorrect.
So options (2) & (3) are correct only.

Q 76. Ans (1) (2) (3) (4)

$p(z)$ is non constant polynomial.

For given $R>0$

$S_R=\{z\in C:|p(z)|<R\}$

As perimage of open set under non constant analytic function is open set and $|p(z)|<R$ is open set.

So, S_R is an open set.
So, option (1) is correct.

Also $|p(z)|<R$ is bounded set

So, S_R is also bounded set.
So, option (2) is correct.
Also, boundary of open set will be mapped into boundary of it's image, so for every z on the boundary of S_R, $|p(z)|=R$

So, option (3) is correct.

As, $|p(z)|<R$, so $p(z)=0$ will satisfy it, so every connected component of S_R contains a zero of $p(z)$.

So option (4) is correct.
Hence all four options are correct.

Q 77. Ans (1) (3)

(1) Set $S=\left\{\frac{2^{2024}}{e}q\mid q\in Q\right\}$ is dense set in R.

i.e. $\tilde{S} = R$, so $\forall\, x, y \in R$, (x, y) contains at least one point of S, else $\tilde{S} \neq R$

Hence there exist $r \in Q$ s.t. $x < \frac{2^{2024}}{e} r < y$

hence option (1) is correct.

(2) If we take $a_n = 2\log n$ then

$\limsup_{n\to\infty} \frac{a_n}{\log n} = \limsup_{n\to\infty} 2 = 2 = L$ but

$\limsup_{n\to\infty} a_n = \limsup_{n\to\infty} 2\log n = \infty$

so option (2) is incorrect.

(3) The set of all finite subsets of Q has number of elements in it which is bounded above,

so cardinality of this set will be $\sum_{i=0}^{n} \left|Q_{C_i}\right|; n$ is fixed natural number.

Now as Q is countably infinite so $\sum_{i=0}^{n} \left|Q_{C_i}\right|$ is also countably infinite.

So, option (3) is correct.

(4) If S is set of all continuous functions from R to $\{0,1\}$ then $S = \{f, g\}$ where

$f(x) = 0\ ;\ \forall\, x \in R\ \&\ g(x) = 1, \forall\, x \in R$

So, $|S| = 2$ i.e. S is finite set , hence option (4) is incorrect.

So, option (1) & (3) are correct only.

Q 78. Ans (2) (4) (Statistics)

Q 79. Ans (1) (2) (Statistics)

Q 80. Ans (1) (2) (3) (4)

If $f : (0,1] \to R$ is monotonic function, so

(1) f is Riemann integrable in $[0,1]$

So, option (1) is correct.

(2) As f is monotonic, so it's set of discontinuity is countable set, so it cannot contain a non-empty open set which is uncountable.

so, option (2) is correct.

(3) F is Riemann integrable, so it is also Lebesgue integrable, & Lebesgue measurable

So, option (3) is correct.

(4) f is monotonic, so f is a Borel measurable function.

So, option (4) is correct.

So, all four options are correct.

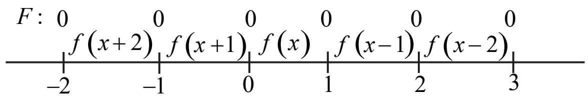
Q 81. Ans (1) (2) (3) (Topology)

Q 82. Ans (1) (2) (3)

$$F(x) = \sum_{n=-\infty}^{\infty} f(x+h)\ ; x \in R$$

$$= \ldots f(x-2) + f(x+1) + f(x) + f(x+1) + f(x+2) + \ldots$$

Also $f(x) = 0$ if $x \leq 0\ \&\ x \geq 1$



$\Rightarrow$ $F(x) = 0\ ;\ x \in I$

$f(x-K); x \in (K, K+1); K \in I$

$\forall\, K \in I$

$\lim_{x\to K^-} F(x) = f(K-(K-1)) = f(1) = 0$

$\lim_{x\to K^+} F(x) = f(K-K) = f(0) = 0$

So, $F(x)$ is continuous at I.

Also at $R \setminus I$ it is obviously continuous.

Also $F(x)$ is continuous periodic function with period 1, so $F(x)$ is uniformly continuous in R.

Also Range $(F(x))$ in R is equal to Range $(F(x))$ in $[0,1]$ so it is bounded.

Q 83. (1) (2) (3) (Markov Chain)

Q 84. Ans (1) (2) (4)

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2} + e^{xy}$$

along family of parabolas $y = mx^2$ becomes

$\lim_{x\to 0} \frac{mx^4}{(1+m^2)x^4} = \frac{m}{1+m^2}$ which is dependent on parameter m, so limit does not exist hence $f(x,y)$ is not continuous at $(0,0)$

so $f(x,y)$ is not differentiable at $(0,0)$, but $f(x,y)$ is differentiable in $R^2 \setminus \{(0,0)\}$.

Also D.D at $(0,0)$ i.e.

$$D_\theta f(0,0) = \lim_{t\to 0^+} \frac{\frac{t^3\cos^2\theta\sin\theta}{t^2\left(t^2\cos^4\theta+\sin^2\theta\right)} + e^{t^2\cos\theta\sin\theta} - 1}{t}$$

$$= \lim_{t\to 0^+} \frac{\cos^2\theta\sin\theta}{\left(t^2\cos^4\theta+\sin^2\theta\right)} + \lim_{t\to 0^+}\frac{e^{t^2\sin\theta\cos\theta}-1}{t} \quad (1)$$

$$\lim_{t\to 0^+}\frac{\cos^2\theta\sin\theta}{t^2\cos^4\theta+\sin^2\theta} = 0; \theta = 0,\pi$$

$$= \frac{\cos^2\theta}{\sin\theta}; \theta \neq 0,\pi$$

for $\lim_{t\to 0^+}\frac{e^{t^2\sin\theta\cos\theta}-1}{t}$ $\quad \left[\frac{0}{0}\text{ case}\right]$

$$= \lim_{t\to 0^+}\frac{e^{t^2\sin\theta\cos\theta}.\,2t\sin\theta\cos\theta}{1} = 0$$

$\forall\,\theta \in [0,2\pi)$

Hence in (1) both part exist, so

All the directional derivatives of f exits at $(0,0)$

So, option (1),(2) & (4) are correct only.

Q 85. Ans (2) (Topology)

Q 86. Ans (2) (3)

$A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ has non-zero entries at $(1,3),(3,2)\&(2,1)$ as 1.

So A^2 has non-zero entries as $1\times 1 = 1$ at $(1,2),(3,1)\&(2,3)$ only.

i.e. $A^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

and A^3 has non-zero entries as 1 at $(1,1),(2,2)\&(3,3)$. So $A^3 = I$.

Hence $A^3 - I = 0 \Rightarrow x^3 - 1 = f(x)$ is an annihilating polynomial of A.

So, minimal polynomial $m(x)$ of A divides x^3-1.

$\Rightarrow \quad m(x) \mid (x^3-1) = (x-1)(x-w)(x-w^2)$

So, $1, w \& w^2$ are possible eigenvalues of A.

Also $tr(A) = 0 \Rightarrow$ eigenvalues of A are $1, w \& w^2$

$\Rightarrow \quad m(x) = (x-1)(x-w)(x-w^2)$.

So, A is diagonalisable over C but not diagonalisable over R.

$B = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

has non-zero entries 1 at $(1,3),(3,2),(2,1)\&$ $(4,4)$

hence $B^3 = I$ & B is real orthogonal matrix, so it is diagonalisable over C.

Also x^3-1 is an annihilating polynomial of B, so again possible eigenvalue of B will be $1, w \& w^2$, so eigenvalues of B will be $1,1,w\&w^2$ so it is not diagonalisable over R.

Q 87. Ans (2) (3) (Statistics)

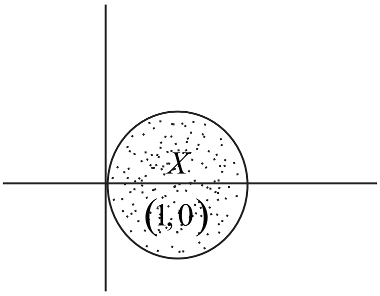
Q 88. Ans (1) (3) (4) (Statistics)

Q 89. Ans (1) (2) (3)

Given $f : C\setminus\{-1,1\} \to C$ is a holomorphic function and range of f do not contain any value from the set.

$S = \{z \in C : |z-1| < 1\}$

i.e.



So by restriction of use of Liouville's theorem $f(z)$ is a constant function.

So, $f(z)$ is bounded and

$\lim_{z\to 1} f(z) \& \lim_{z\to 1} f(z)$ both exist, so $f(z)$ has removable singularities at $-1 \& 1$.

So, option 1,2 & 3 are correct only.

Q 90. Ans (1) (2)

Recall the definition.

G is said to be divisible if for every $y \in G$ and every positive integer n. $\exists\ x \in G$ such that $x^n = y$.

so, options (1) & (2) are true

Let $y \in Q$, and for each $n \in \mathbb{N}$ we have,

$x^n = y \Rightarrow nx = y$

$\Rightarrow x = \frac{y}{n} \in Q$

Clearly, $x = \frac{y}{n} \in Q$ for each $n \in \mathbb{N}$

$\Rightarrow$ For each $y \in Q$ and each $n \in \mathbb{N}$

$\exists x \in Q$ such that $x^n = y$

$\Rightarrow$ $(Q,+)$ is divisible group

Option (3) and (4) are not true.

$\mathbb{Z}_n$ for $n > 1$ is not divisible..

If we take $\tau = (1\ 2\ 3\ 4) \in S_5$ then for $n = 2$,

$\nexists\, x \in S_5$ such that $x^2 = (1\ 2\ 3\ 4)$

Since x^2 is always even permutation for all $x \in S_5$ but $(1\ 2\ 3\ 4)$ is an odd permutation.

So, options 1 & 2 are correct only.

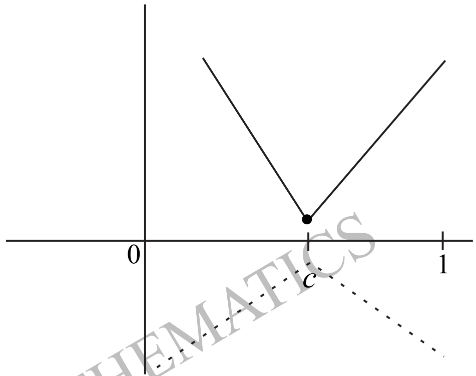
Q 91. Ans (3) (4)

$S(b)$ is set of all broken extremals with one corner of the variational problem,

minimize $J[y] = \int_0^1 \left((y')^4 - 3(y')^2 \right) dx$

subject to $y(0) = 0; y(1) = 6$.

It should have jump discontinuity at point $c \in (0,1)$



Here $F(x, y, y') = (y')^4 - 3(y')^2$ which is independent of $x \& y$.

So, it's solution will be

$$y(x) = \begin{cases} m_1 x + C_1\ ;\ 0 \le x < C \\ m_2 x + C_2\ ;\ C \le x \le 1 \end{cases}$$

where $y(0) = 0$ & $y(1) = b$

$\Rightarrow$ $m_1 .0 + C_1 = 0 \Rightarrow C_1 = 0$

$y(1) = b \Rightarrow m_2 + C_2 = b \Rightarrow C_2 = b - m_2$

$$\Rightarrow \quad y(x) = \begin{cases} m_1 x & ;\ 0 \le x < C \\ m_2(x-1) + b\ ; & C \le x \le 1 \end{cases} \qquad (1)$$

where at $x = c$

(i) y should be continuous

(ii) F should be continuous

(iii) $\frac{\partial F}{\partial y'}$ should be continuous

For (i) Continuity of y at C

$m_1 C = m_2 (C - 1) + b$

$\Rightarrow$ $(m_1 - m_2) C = -m_2 + b$

$\Rightarrow \quad C = \dfrac{b - m_2}{m_1 - m_2} \quad \because C \in (0,1)$

$\therefore m_1 - m_2 \neq 0$

$\Rightarrow m_1 \neq m_2$

For slopes of lines

$F(y') = y'^4 - 3y'^2 = 0 \Rightarrow m^4 - 3m^2 = 0$ (2)

$\dfrac{\partial F}{\partial y'} = 4y'^3 - 6y' = 0 \Rightarrow 4m^3 - 6m = 0$ (3)

From (2) $m_1^4 - 3m_1^2 = m_2^4 - 3m_2^2$

$\Rightarrow \quad m_1^4 - m_2^4 = 3(m_1^2 - m_2^2)$

$\Rightarrow \quad (m_1^2 - m_2^2)(m_1^2 + m_2^2 - 3) = 0$

$\Rightarrow \quad m_1^2 + m_2^2 = 3$ or $m_1 = \pm m_2$

i.e. $m_1 = -m_2$ or $m_1^2 + m_2^2 = 3$ (4)

From (3) $4m_1^3 - 6m_1 = 4m_2^3 = 6m_2$

$\Rightarrow \quad 4(m_1^3 - m_2^3) = 6(m_1 - m_2)$

$\Rightarrow \quad 2(m_1 - m_2)(m_1^2 + m_2^2 + m_1 m_2) = 3(m_1 - m_2)$

$\Rightarrow \quad (m_1 - m_2)\left(m_1^2 + m_2^2 + m_1 m_2 - \dfrac{3}{2}\right) = 0$

$\Rightarrow \quad m_1 = m_2$ (not possible) , so

$m_1^2 + m_2^2 + m_1 m_2 = \dfrac{3}{2}$ (5)

From (4) & (5)

$m_1 = -m_2 \Rightarrow m_1^2 = \dfrac{3}{2} \Rightarrow m_1 = \pm\dfrac{\sqrt{3}}{2}$

So , $m_1 = \dfrac{\sqrt{3}}{2} \Rightarrow m_2 = -\dfrac{\sqrt{3}}{2}$

& $m_1 = -\dfrac{\sqrt{3}}{2} \Rightarrow m_2 = \dfrac{\sqrt{3}}{2}$

Also, $m_1^2 + m_2^2 = 3$ & $m_1^2 + m_2^2 + m_1 m_2 = \dfrac{3}{2}$

$\Rightarrow \quad m_1 m_2 = -\dfrac{3}{2}$

$\Rightarrow \quad (m_1 + m_2)^2 = 3 + 2\left(\dfrac{-3}{2}\right) = 0$

$\Rightarrow \quad m_1 + m_2 = 0 \Rightarrow m_1 = -m_2$

So, $m_1 = \dfrac{\sqrt{3}}{2} \Rightarrow m_2 = -\dfrac{\sqrt{3}}{2}$

& $m_1 = -\dfrac{\sqrt{3}}{2} \Rightarrow m_2 = \dfrac{\sqrt{3}}{2}$

Also , $c = \dfrac{b - m_2}{m_1 - m_2} \in (0,1)$

$\Rightarrow \quad 0 < \dfrac{b - m_2}{m_1 - m_2} < 1 \Rightarrow m_2 < b < m_1$

or $0 < \dfrac{m_2 - b}{m_2 - m_1} < 1 \Rightarrow m_1 < b < m_2$

Hence $-\dfrac{\sqrt{3}}{2} < b < \dfrac{\sqrt{3}}{2}$ (6)

If $b = 2$ then condition (6) is not satisfied

So, $S(b)$ is empty set

Hence option (1) is incorrect and option (3) is correct.

If $b = \dfrac{1}{2}$ then it satisfy condition (6) and in this case

(i) If $m_1 = \dfrac{\sqrt{3}}{2}$ then broken extremal is

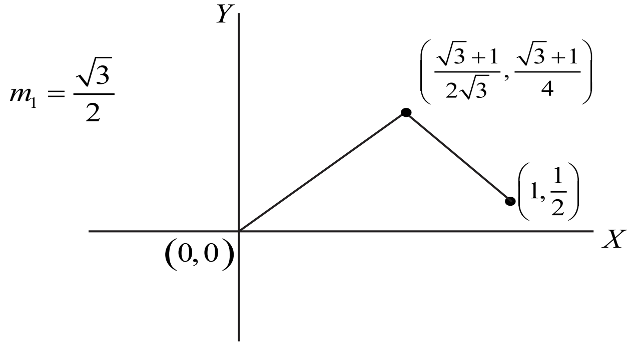
$$y = \begin{cases} \dfrac{\sqrt{3}}{2}x & ; 0 \le x \le \dfrac{\sqrt{3}+1}{2\sqrt{3}} \\ -\dfrac{\sqrt{3}}{2}(x-1) + \dfrac{1}{2}; & \dfrac{\sqrt{3}+1}{2\sqrt{3}} \le x \le 1 \end{cases}$$

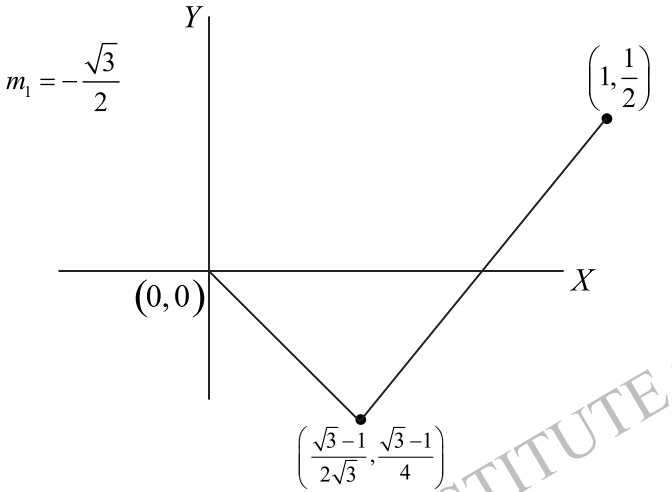
(ii) If $m_1 = \dfrac{-\sqrt{3}}{2}$ then broken extremal is

$$y = \begin{cases} -\dfrac{\sqrt{3}}{2}x & ; 0 \le x \le \dfrac{\sqrt{3}-1}{2\sqrt{3}} \\ \dfrac{\sqrt{3}}{2}(x-1) + \dfrac{1}{2}; & \dfrac{\sqrt{3}-1}{2\sqrt{3}} \le x \le 1 \end{cases}$$

Hence $S\left(\dfrac{1}{2}\right) = 2$

So option (2) is incorrect and option (4) is correct. Also diagram of broken extremals are





So, options 3 & 4 are correct only.

Q 92. Ans (4)

Maximise $P = x_1 + x_2$ (1)

S.t. $x_1 + x_2 + 2x_3 \le 5$ (2)

$2x_1 - 3x_3 \ge 1$ (3)

$x_2 + x_3 \le 0$ (4)

$x_1 \ge 0, x_2 \ge 0 \,\&\, x_3 \ge 0$ (5)

from (4) & (5) we get $x_2 = x_3 = 0$

So eq- (3) becomes $x_1 \ge 1/2$ & eq-(2) becomes $x_1 \le 5$

$\Rightarrow$ In feasible region

$$S = \left\{(x_1, x_2, x_3) \mid \frac{1}{2} \le x_1 \le 5, x_2 = x_3 = 0\right\}$$

$$\Rightarrow \quad S = \left\{(x, 0, 0) \mid x \in \left[\frac{1}{2}, 5\right]\right\}$$

So, optimal point is $(5,0,0)$ & Maximum value of $P = 5 + 0 = 5$

i.e. optimal solution is 5.

Also corner points of Feasible region are $\left(\frac{1}{2}, 0, 0\right) \& (5,0,0)$.

So option (4) is correct only.

Q 93. Ans (3) (4)

Given $\frac{dx}{dt} = x - 4e^{-2t}\, y$ (1)

$\frac{dy}{dt} = e^{2t}\, x - y$ (2)

$x(0) = 1,\ y(0) = 1$ (I.C.)

From (2) $\frac{d^2y}{dt^2} = e^{2t}\frac{dx}{dt} + 2e^{2t}x - \frac{dy}{dt}$

From (1) $\frac{d^2y}{dt^2} = e^{2t}\left(x - 4e^{-2t}y\right) + 2e^{2t}x - \frac{dy}{dt}$

$\Rightarrow \quad \frac{d^2y}{dt^2} = 3xe^{2t} - 4y - \frac{dy}{dt}$

$\Rightarrow \quad \frac{d^2y}{dt^2} = 3\left(\frac{dy}{dt} + y\right) - \frac{dy}{dt} - 4y$

$\Rightarrow \quad \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = 0$

$\Rightarrow \quad \left(D^2 - 2D + 1\right)y = 0 \Rightarrow (D-1)^2\, y = 0$

$\Rightarrow \quad y = C_1e^t + C_2t\,e^t$ (3)

$\Rightarrow \quad \frac{dy}{dt} = C_1e^t + C_2e^t + tC_2\,e^t$

$\Rightarrow \quad x = \left(y + \frac{dy}{dt}\right)e^{-2t}$

$\Rightarrow \quad x = \left(2C_1e^t + 2C_2te^t + C_2e^t\right)e^{-2t}$

$\Rightarrow \quad x = 2C_1e^{-t} + 2C_2t\,e^{-t} + C_2e^{-t}$ (4)

So, $y(0) = 1 \Rightarrow C_1 = 1$

& $x(0) = 1 \Rightarrow 2C_1 + C_2 = 1 \Rightarrow C_2 = -1$

$\Rightarrow \quad x(t) = e^{-t} - 2t\,e^{-t}$ & $y(t) = e^t - t\,e^t$

Now, $\lim_{t\to\infty} t^{-2}x(t)y(t) =$

$\lim_{t\to\infty} t^{-2}\left(e^{-t}-2te^{-t}\right)\left(e^{t}-te^{t}\right)$

$= \lim_{t\to\infty}\frac{1}{t^2}\left(1-t-2t+2t^2\right)=2$

So, option (4) is correct & option (1) is incorrect.

Also $x(1)=-e^{-1}$; $x(1/2)=0$

$y(1/2)=\frac{1}{2}e^{1/2}$ & $y(1)=0$

so option (3) is correct & option (2) is incorrect.

So, options 3 & 4 are correct only.

Q 94. Ans (1) (3) (Statistics)

Q 95. Ans (1) (3)

$\forall n\geq 2$, $T:C^n\to C^n$ is a C-linear transformation.

Also V is a subspace of C^n such that $T(V)\subseteq V$

(1) Let basis of V be $B_v=\{\alpha_1,\alpha_2,...,\alpha_m\}$ (i) which is linearly independent set of vectors from C^n, so it can be extended to form a basis of C^n by extension theorem, so let basis of C^n be $B_{C^n}=\{\alpha_1,\alpha_2,...,\alpha_m,\beta_1,\beta_2,...,\beta_{n-m}\}$

$\because$ $\dim\left(C^n(C)\right)=n$ (ii)

Let $W=L\left(\{\beta_1,\beta_2,...,\beta_{n-m}\}\right)$ then

$B_W=\{\beta_1,\beta_2,...,\beta_{n-m}\}$ will be basis of subspace W of C^n.

Also for this choice of subspace W,

$C^n=V+W$ & $V\cap W=\{0\}$

As $C^n=V\oplus W$

So option (1) is correct.

(2) If we take $T:C^2\to C^2$ as

$T(Z_1,Z_2)=(2Z_1,3Z_1+4Z_2);n=2$ then

$T(0,1)=(0,4)$ & $T(1,0)=(2,3)$

Now if $V=L\left(\{(0,1)\}\right)$ then

$T(V)=\{K(0,4)\mid K\in C\}\subseteq V$ but if we want $C^2=V+W$ & $V\cap W=\{0\}$ then W can be $L\left(\{(1,0)\}\right)$ but $T(W)=L\left(\{(2,3)\}\right)$ so $T(W)$ is not subset of W.

so, option (2) is incorrect.

(3) If $\exists K\in N$ s.t. $T^K=I$ then define an inner product on C^n and take $W=V^{\perp}$ i.e. orthogonal complement of V.

Since $T^K=I$ so T is invertible.

As we know that for $w\in W$ and $v\in V$, we have $\langle T(w),v\rangle=\langle w,T^{-1}(v)\rangle$

Since $T(V)\subseteq V$ & $T^K=I, T^{-1}(v)\subseteq V$

$\Rightarrow$ $\langle T(w),V\rangle=\langle w,T^{-1}(v)\rangle=0$

$\because$ $T^{-1}(V)\subseteq V$

$\Rightarrow$ $\langle T(w),V\rangle=0 \Rightarrow T(w)\subset W$

$\Rightarrow$ $T(w)\subseteq W$

Also $C^n=V\oplus W$

So option (3) is correct.

(4) If we take $n=2$ i.e. $C^n=C^2$ & If $V=L\left(\{(1,0)\}\right)$ & $W=L\left(\{(0,1)\}\right)$ and $T(\alpha)=2\alpha$ then $T(V)=L\left(\{(2,0)\}\right)\subseteq V$ & $T(W)=L\left(\{(0,2)\}\right)\subseteq W$ & $C^2=V\oplus W$ i.e. $C^2=V+W$ & $V\cap W=\{0\}$.

$\therefore$ $T=2I$ $\therefore \nexists K\in N$ s.t. $T^K=I$

so option (4) is incorrect.

Hence options (1) & (3) are correct only.

Q 96. Ans (1) (2) (3)

Given I.B.V.P is

$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2};(x,t)\in(0,1)\times(0,\infty)$

$u(x,0)=4x(1-x)$; $x\in[0,1]$

$u(0,t)=u(1,-t)=0$; $t\geq 0$

It is Heat equation of finite length

So, it's solution is

$$u(x,t)=\sum_{n=1}^{\infty} D_n \sin n\pi x . e^{-Kn^2\pi^2 t} \quad (1)$$

So $\lim_{t\to\infty} u(x,t)=0 \because \lim_{t\to\infty} e^{-Kn^2\pi^2 t}=0$

So option (1) is correct.

From (1)

$$u(x,0)=\sum_{n=1}^{\infty} D_n \sin n\pi = 4x(1-x) \quad (2)$$

$$\Rightarrow \quad u(1-x,0)=\sum_{n=1}^{\infty} D_n \sin n\pi(1-x)=4(1-x)x \quad (3)$$

$$\Rightarrow \quad u(1-x,0)=\sum_{n=1}^{\infty}(-1)^{n+1} D_n \sin n\pi x$$

From (2) & (3)

$u(x,0)=u(1-x,0)=0\Rightarrow$

$$\sum_{n=1}^{\infty}(-1)^{n+1} D_n \sin n\pi = \sum_{n=1}^{\infty} D_n \sin n\pi x$$

$\Rightarrow$ $n+1$ should be even i.e. n should be odd, so

$$u(x,t)=\sum_{n=1}^{\infty} D_{2n-1}\sin(2n-1)\pi x\, e^{-K(2n-1)^2\pi^2 t}$$

and hence $u(x,t)=u(1-x,t)$

So, option (2) is correct.

Now $f(t)=\int_0^1 (u(x,t))^2 dx$

$$\Rightarrow \quad f'(t)=\int_0^1 \frac{\partial}{\partial t}\left(u^2(x,t)\right)dx$$

$$=\int_0^1 2u(x,t)u_t(x,t)\,dx$$

$$=2\int_0^1 u\,u_{xx}\,dx = 2u\,u_x\Big|_0^1 - 2\int_0^1 u_x^2 dx$$

(Integration by parts)

$$\Rightarrow \quad f'(t)=2u(1,t)u_x(1,t)-2u(0,t)u_x(0,t)$$

$$-2\int_0^1 u_x^2\,dx$$

$$=-2\int_0^1 u_x^2\,dx \qquad \because u(0,t)=u(1,t)=0$$

$\Rightarrow$ $f'(t)\le 0$, so $f(t)$ is decreasing function i.e. non increasing function.

So option (3) is correct and option (4) is incorrect.

Hence options (1) , (2) & (3) are correct only.

Q 97. Ans (1) (2)

Given volterra integral equation is

$$u(x)=3+\sin x+\int_0^x \frac{3+\sin x}{3+\sin t}u(t)\,dt \quad (1)$$

Here Kernel, $K(x,t)=\dfrac{3+\sin x}{3+\sin t}$

$f(x)=3+\sin x$; $\lambda=1$

Here we have

(i) $K(x,x)=1$

(ii) $K(x,t)\times K(t,x)=1$

Resolvent Kernel,

$$R(x,t)=e^{\lambda(x-t)}K(x,t)=e^{x-t}\left(\frac{3+\sin x}{3+\sin t}\right)$$

and it's solution will be

$$u(x)=f(x)+\lambda\int_0^x R(x,t)f(t)\,dt$$

$$=3+\sin x+\int_0^x e^{x-t}\left(\frac{3+\sin x}{3+\sin t}\right)(3+\sin t)\,dt$$

$$\Rightarrow \quad u(x)=3+\sin x+\int_0^x e^x(3+\sin x)e^{-t}\,dt$$

$$=3+\sin x+e^x(3+\sin x)\left(-e^{-t}\right)\Big|_0^x$$

$$=(3+\sin x)\left(1+e^x\left(1-e^{-x}\right)\right)$$

$$=(3+\sin x)\left(1+e^x-1\right)$$

$$\Rightarrow \quad u(x)=(3+\sin x)e^x \quad (2)$$

(1) $u\left(\frac{\pi}{2}\right)=4e^{\pi/2}$ (correct)

(2) $u(\pi)=3e^{\pi}$ (correct)

(3) $u(-\pi)=3e^{-\pi}$ (incorrect)

(4) $u\left(-\frac{\pi}{2}\right)=2e^{-\pi/2}$ (incorrect)

Hence options (1) & (2) are correct only.

Q 98. Ans (1) (4)

A polynomial $f(x)\in\mathbb{F}[x]$ is said to be separable if it has no repeated root in any extension of $\mathbb{F}$.

Since $f'(x) = 2025x^{2024} - 1 = 0$ over $\mathbb{F}_5$

$\Rightarrow$ $f(x)$ is not separable polynomial over any extension of $\mathbb{F}_5$

$\Rightarrow$ $f(x)$ has repeated roots in algebraic closure of $\mathbb{F}_5$,

Now $f(x) = x^{2025} - 1 = \left(x^{81}\right)^{25} - 1$

$\Rightarrow$ $f(x)$ has separable factor of degree 81 in algebraic closure of $\mathbb{F}_5$.

$\Rightarrow$ S has exactly of 81- elements in algebraic closure of $\mathbb{F}_5$

$\Rightarrow$ $f(x)$ has 81-roots in algebraic closure of $\mathbb{F}_5$

Option (4) is correct.

Since S is containg 81st roots of unity.

$\Rightarrow$ S is cyclic group of order 81 or $S \approx \mathbb{Z}_{81}$.

$\Rightarrow$ Option (1) is correct.

Hence options (1) & (4) are correct only.

Q 99. Ans (3) (4)

A is positive definite matrix of order 4, so A is congruent to I_4, so by considering $A = I_4$ we get

$$B = \begin{pmatrix} 0 & a & b & c & d \\ a & 1 & 0 & 0 & 0 \\ b & 0 & 1 & 0 & 0 \\ c & 0 & 0 & 1 & 0 \\ d & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \quad |B| = \begin{vmatrix} 0 & a & b & c & d \\ a & 1 & 0 & 0 & 0 \\ b & 0 & 1 & 0 & 0 \\ c & 0 & 0 & 1 & 0 \\ d & 0 & 0 & 0 & 1 \end{vmatrix}$$

expand it along last row.

$$= 1.\begin{vmatrix} 0 & a & b & c \\ a & 1 & 0 & 0 \\ b & 0 & 1 & 0 \\ c & 0 & 0 & 1 \end{vmatrix} + d\begin{vmatrix} a & b & c & d \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

$$= -d^2 + \begin{vmatrix} 0 & a & b & c \\ a & 1 & 0 & 0 \\ b & 0 & 1 & 0 \\ c & 0 & 0 & 1 \end{vmatrix}$$

$= -d^2 - c^2 - b^2 - a^2 = -\left(a^2 + b^2 + c^2 + d^2\right)$ by recursion.

So,

$\forall a,b,c,d \in R,\ |B| = -\left(a^2 + b^2 + c^2 + d^2\right) < 0$

So, option (3) & (4) are correct.

Q 100. Ans (2) (4)

$f_n(x) = nx(1-x)^n$ (Given)

so it's pointwise limit function $f(x)$ in $[0,1]$ will be $f(x) = \lim\limits_{n\to\infty} f_n(x) = \lim\limits_{n\to\infty} n\,x(1-x)^n$

$= 0\ ;\ \forall x \in [0,1]$

So, $(f_n)_{n\ge1}$ converges pointwise to a continuous function on $[0,1]$.

So, option (1) is incorrect and option (2) is correct, also option (3) is incorrect.

Now for U.C.

$\left|f_n(x) - f(x)\right| = \left|nx(1-x)^n - 0\right| =$

$n\,x(1-x)^n = g(x)$ say

Method 1:-

$\because \quad g\left(\frac{1}{n}\right) = n.\frac{1}{n}\left(1-\frac{1}{n}\right)^n = \left(1-\frac{1}{n}\right)^n$

$\therefore \quad \lim\limits_{n\to\infty} g\left(\frac{1}{n}\right) = \lim\limits_{n\to\infty}\left(1-\frac{1}{n}\right)^n =$

$\left(1^\infty \text{ case}\right)\ e^{\lim\limits_{n\to\infty} n\left(1-\frac{1}{n}-1\right)} = e^{-1} = \frac{1}{e}$

$\Rightarrow \quad M_n = \sup\limits_{x\in[0,1]}\left|f_n(x) - f(x)\right| \ge g\left(\frac{1}{n}\right)$ gives

$\lim\limits_{n\to\infty} M_n \ge \lim\limits_{n\to\infty} g\left(\frac{1}{n}\right) = \frac{1}{e}$

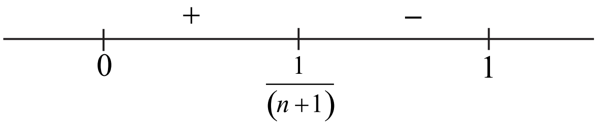
$\Rightarrow \quad \lim\limits_{n\to\infty} M_n \ne 0$, so convergence is not uniform.

Method 2:-

$$g'(x) = n(1-x)^n + n\,x.n(1-x)^{n-1}(-1)$$

$$= n(1-x)^{n-1}\left[(1-x) - nx\right]$$

$$= -n(n+1)(1-x)^{n-1}\left[x - \frac{1}{(n+1)}\right]$$



sign scheme of $g'(x)$

$$\Rightarrow \quad \sup_{x\in(0,1]}\left(g(x)\right) = M_n = g\left(\frac{1}{n+1}\right)$$

$$\Rightarrow \quad M_n = \frac{n}{n+1}\left(1 - \frac{1}{n+1}\right)^n$$

$$\& \lim_{n\to\infty} M_n = \lim \frac{n}{n+1} \lim\left(1 - \frac{1}{n+1}\right)^n$$

$$= 1 \times e^{\lim n\left(1 - \frac{1}{n+1} - 1\right)}$$

$$= e^{-1} = \frac{1}{e} \neq 0$$

So convergence is not uniform.
So option (2) & (4) are correct only.

Q 101. Ans (1) (2) (4)

Option (1) is correct,

Since $\phi(n)$ is even for all $n \geq 3$

Option (2) is correct,

Since $|S| > \frac{|G|}{2}$

(By using closure property)
Option (3) is not correct,

Since $\mathbb{R}[x_1, x_2, .., x_n]$ is not P.I.D.

Hence $\mathbb{R}[x_1, x_2, .., x_n]$ is not E.D.

Because the ideal $\langle x_1, x_2 \rangle$ is not generated by single element in $\mathbb{R}[x_1, x_2, ..., x_n]$

Option (4) is correct,

The subset $\left\{ f \in c[0,1] \mid f\left(\frac{1}{2}\right) = 0 \right\}$ is collection of one point vanishing functions in $c[0,1]$.

Hence the given set is prime and maximal ideal.
so, options (1), (2) and (4) are correct.

Q 102. Ans (2) (3) (4)

Given $f = p_1, p_2, ..., p_n \in \mathbb{R}[x]$ and $p_i, 1 < i \leq n$ are irreducible monic polynomial

$\Rightarrow \quad \gcd(p_i, p_j) = 1,\ i \neq j$

By Chinese remainder theorem for ideals we have $\frac{\mathbb{R}[x]}{\langle f \rangle} \approx \frac{\mathbb{R}[x]}{\langle p_1 p_2 ... p_n \rangle} \approx$

$\frac{\mathbb{R}[x]}{\langle p_1 \rangle} \times \frac{\mathbb{R}[x]}{\langle p_2 \rangle} \times ... \times \frac{\mathbb{R}[x]}{\langle p_n \rangle}$

Since $\langle p_i \rangle, 1 \leq i \leq n$ are irreducible monic polynomial

$\Rightarrow \quad \frac{\mathbb{R}[x]}{\langle p_i \rangle} \approx \mathbb{R}$ or $\mathbb{C}$

Hence $\frac{\mathbb{R}[x]}{\langle f \rangle}$ is isomorphic to either finite products of $\mathbb{R}$ or $\mathbb{C}$ or $\mathbb{R}$ and $\mathbb{C}$

$\Rightarrow \quad \frac{\mathbb{R}[x]}{\langle f \rangle}$ is not field.

so, option (1) is not correct. but option (3) is correct.

we know that $\frac{\mathbb{R}[x]}{\langle f \rangle}$ is a finite dimensional $\mathbb{R}$ vector space.

$\Rightarrow$ Option (2) is true
Option (4) is correct.

as, $\nexists\, x \in \frac{\mathbb{R}[x]}{\langle f \rangle}$ such that $x^n = 0$ for some $n \geq 1$

so,option (2), (3) and (4) are correct.

Q 103. Ans (1) (2) (3)

$g(x)$ is continuous function and

$$f(x)=\int_0^x (x-t)\,g(t)\,dt \qquad (1)$$

$$\Rightarrow \quad f'(x)=\int_0^x g(t)\,dt \qquad (2)$$

$$\Rightarrow \quad f''(x)=g(x) \qquad (3)$$

by Leibnitz rule of derivative of integral.

From (1) $f(0)=0$

From (2) $f'(0)=0$ & it exist

From (3) $f''(0)=g(0)$ & it exist

So, options (1), (2) & (3) are correct only.

Q 104. Ans (2) (3)

Given B.V.P (Laplace equation) is

$u_{xx}+u_{yy}=0;(x,y)\in(0,1)\times(0,1)$

$u(x,0)=e^{\pi x}, u(x,1)=-e^{\pi x}; x\in[0,1]$ (1)

$$\left.\begin{aligned} u(0,y)&=\cos(\pi y)+\sin(\pi y); y\in(0,1] \\ u(1,y)&=e^{\pi}\left(\cos(\pi y)+\sin(\pi y)\right); y\in[0,1] \end{aligned}\right\} \qquad (2)$$

From (2) at $x=1$ we get factor e^{π} which at $x=0$ is equal to 1.

So it's contribution is $e^{\pi x}$ & by using (2) we get it's general solution as

$u(x,y)=e^{\pi x}\left[\cos(\pi y)+\sin(\pi y)\right]$ (3)

For $(x_0,y_0)\in(0,1)\times(0,1)$

(i) If $x_0\in(0,1)$ then $e^{\pi x}$ is S.I. function

so, $e^{\pi .0}<e^{\pi x}<e^{\pi .1}$

$\Rightarrow \quad 1<e^{\pi x}<e^{\pi}$ (4)

& $-\sqrt{1^2+1^2}<\cos\pi y+\sin\pi y<\sqrt{1^2+1^2}$

$\Rightarrow \quad -\sqrt{2}\le\cos\pi y+\sin\pi y\le\sqrt{2}$ (5)

From (4) & (5) we get

$\Rightarrow \quad -\sqrt{2}<u(x,y)<\sqrt{2}\,e^{\pi}$

Now $-1\,\&\,e^{\pi}$ lies in this range but $\sqrt{2}\,e^{\pi}$ & $-e^{\pi}$ do not lie in this range.

So, options (2) & (3) are correct only.

Q 105. Ans (1) (3) (4) (Statistics)

Q 106. Ans (2)

Q 107. Ans (1) (3) (Statistics)

Q 108. Ans (1)

$a_n=(-1)^n\left(1+e^{-n}\right)$

$\Rightarrow \quad \lim\limits_{n\to\infty} a_n=\lim\limits_{n\to\infty}(-1)^n\left(1+e^{-n}\right)$

$=\lim\limits_{n\to\infty}(-1)^n(1+0)=\lim(-1)^n=\{1,-1\}$

so $(a_n)_{n\ge1}$ oscillates finitely between $1\,\&-1$.
It do not converge so option (1) is correct.

Now $a_1=-\left(1+\frac{1}{e}\right), a_2=\left(1+\frac{1}{e^2}\right)$

$a_3=-\left(1+\frac{1}{e^3}\right),\ldots$

So magnitude is decreasing but their sign is alternate

Further $b_n=\max(a_1,a_2,\ldots,a_n)$

$\Rightarrow \quad b_1=\max(a_1)=-\left(1+\frac{1}{e}\right)$

& $b_2=b_3=\ldots=\left(1+\frac{1}{e^2}\right)$

so, $\lim\limits_{n\to\infty} b_n=1+\frac{1}{e^2}$ (1)

Also $C_n=\min(a_1,a_2,\ldots,a_n)$

$\Rightarrow \quad C_n=-\left(1+\frac{1}{e}\right); \forall n\in N$

$\Rightarrow \quad \lim C_n=-\left(1+\frac{1}{e}\right)$ (2)

From equations (1) & (2) options 2,3, & 4 are incorrect.
Hence option (1) only is correct.

Q 109. Ans (1) (2) (3) (Statistics)

Q 110. Ans (1)

$V=L\left(\{1,x,x^2,x^3\}\right)$

$D:V\to V$ is given by

$D\left(ax^3+bx^2+cx+d\right)=3ax^2+2bx+c$ (1)

& $M : V \to V$ is given by

$M\left(ax^3+bx^2+cx+d\right)=3ax^3+2bx^2+cx$ (2)

$\Rightarrow$ $DM\left(ax^3+bx^2+cx+d\right)=$

$D\left(3ax^3+2bx^2+cx\right)=9ax^2+4bx+c$ (3)

$MD\left(ax^3+bx^2+cx+d\right)=$

$M\left(3ax^2+2bx+c\right)=6ax^2+2bx$ (4)

From (3) & (4) $DM \neq MD$
so, option (1) is correct.

From (1) & (2)

$\left(D+M\right)\left(ax^3+bx^2+cx+d\right)=3ax^3+$

$\left(3a+2b\right)x^2+\left(2b+c\right)x+c$

Matrix of D is

$$\begin{array}{cccc} D(1) & D(x) & D(x^2) & D(x^3) \end{array}$$

$$D=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix of M is

$$\begin{array}{cccc} M(1) & M(x) & M(x^2) & M(x^3) \end{array}$$

$$M=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$\Rightarrow$ Matrix of D + M

$$D+M=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

which is singular, so option (2) is false.

$$DM=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

which is again singular.
so option (3) is false.

Also Rank $\left(DM\right)=3$

Now matrix of MD is

$$MD=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So, $\rho\left(MD\right)=2$

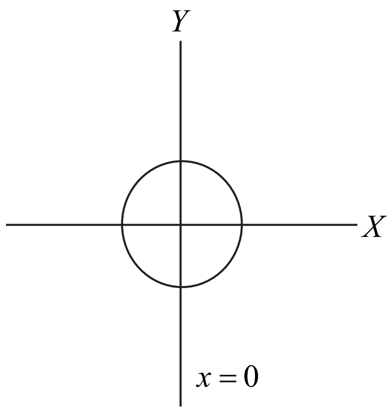
Hence $\rho\left(DM\right)\neq\rho\left(MD\right)$

so option (4) is false
Hence, option (1) is correct only.

Q 111. Ans (3) (4) (Statistics)

Q 112. Ans (1) (2) (4)

$\lim\limits_{(x,y)\to(0,0)} f\left(x,y\right)=\lim\limits_{(x,y)\to(0,0)} 0$ along y-axis



$=\lim\limits_{(x,y)\to(0,0)} \dfrac{x}{|x|}\sqrt{x^2+y^2}$, elsewhere

$=\lim\limits_{r\to 0}\dfrac{r\cos\theta}{|r\cos\theta|}.\ r=\lim\limits_{r\to 0} \pm r=0$

so $\lim\limits_{(x,y)\to(0,0)} f\left(x,y\right)=0$

$\because \quad \lim_{(x,y)\to(0,0)} f(x,y) = 0 = f(0,0)$

$\therefore \quad f(x,y)$ is continuous at $(0,0)$

Now $f_x(0,0) = \lim_{h\to 0} \dfrac{f(h,0)-f(0,0)}{h}$

$$= \lim_{h\to 0} \frac{\frac{h}{|h|}\sqrt{h^2}-0}{h} = 1$$

& $f_y(0,0) = \lim_{K\to 0} \dfrac{f(0,K)-f(0,0)}{K}$

$$= \lim_{K\to 0} \frac{0-0}{K} = 0$$

So, the derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ exist at $(0,0)$

For differentiability at $(0,0)$

$$df = f(h,K) - f(0,0) = f_x(0,0).h + f_y(0,0).K + g(h,K)\sqrt{h^2+K^2}$$

$\Rightarrow \quad f(h,K) = 1.h + \sqrt{h^2+K^2}\ g(h,K)$

$\Rightarrow \quad g(h,K) = \dfrac{f(h,K)-h}{\sqrt{h^2+K^2}}$

$\Rightarrow \quad g(h,K) = \dfrac{0-0}{\sqrt{0^2+K^2}}$ if $h=0$

$$= \frac{\frac{h}{|h|}\sqrt{h^2+K^2}-h}{\sqrt{h^2+K^2}}, \text{ otherwise}$$

$$= \frac{h\left(\sqrt{h^2+K^2}\right)-h|h|}{|h|\sqrt{h^2+K^2}}$$

$$\lim_{(h,K)\to(0,0)} g(h,K) = \lim_{r\to 0} \frac{r^2\left(\cos\theta - \cos^2\theta\right)}{r^2.(\cos\theta)}$$

(for second part)

$= \lim_{r\to 0}\left(1-|\cos\theta|\right)$, so it does not exist

So $f(x,y)$ is not differentiable at $(0,0)$.

Hence, options (1),(2) & (4) are correct only.

Q 113. Ans (1) (3) (4) (Statistics)

Q 114. Ans (1) (3)

$T: M_2(R) \to M_2(R)$ is linear transformation given by $T(X) = AXB^t$;

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}; B = \begin{pmatrix} -1 & 0 \\ 1 & 5 \end{pmatrix}$$

& let $X = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}$

$$\Rightarrow \quad T\left(\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}\right) = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} x_1+2x_3 & x_2+2x_4 \\ 3x_3 & 3x_4 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -x_1-2x_3 & x_1 5x_2+2x_3+10x_4 \\ -3x_3 & 3x_3+15x_4 \end{pmatrix}$$

So matrix of linear transformation T is

$$T = \begin{bmatrix} -1 & 0 & -2 & 0 \\ 1 & 5 & 2 & 10 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 3 & 15 \end{bmatrix}$$

$\Rightarrow \quad trace(T) = -1+5+(-3)+15 = 16$

and $\det(T) = \begin{vmatrix} -1 & 0 \\ 1 & 5 \end{vmatrix} . \begin{vmatrix} -3 & 0 \\ 3 & 15 \end{vmatrix} = (-5)(-45)$

$$= 225$$

So, options (1) & (3) are correct only.

Short cut

$$\det(T) = (\det(A))^2\left(\det(B^t)\right)^2$$

$$= (3)^2(-5)^2 = 9\times 25 = 225$$

& $trace(T) = (2tr(A)) + 2(tr(B))$

$$= (2\times 4) + (2\times 4)$$

$$= 8 + 8 = 16.$$

Hence, options (1) & (3) are correct only.

Q 115. Ans (1) (2)

Given that $f(z)$ is holomorphic function in

$D=\{z\in c:|z|<1\}$ and $g(z)=e^{\frac{1}{z}}f(z)$ is bounded in $D\setminus\{0\}$ so $\lim_{z\to 0}e^{\frac{1}{z}}f(z)$ must be bounded which is possible only if it is a deleted nbd of 0.

$f(z)=0$ and this means that $f(z)$ has uncountable number of zeros in D, so, $f(z)=0;\forall z\in D$ as zeroes of non constant analytic functions are isolated and hence $f(0)=0$.

So, options (1) & (2) are correct only.

Q 116. Ans (1) (3) (4) (Statistics)

Q 117. Ans (2) (3)

$X(n)=\{x\in X \mid x\le n\}$ and $\lim_{n\to\infty}\frac{|X(n)|}{n}=1$

$\Rightarrow$ $X(n)$ is countably infinite set.

Also $X_1,X_2,\ldots,X_8$ are pairwise disjoint set such that $\bigcup_{i=1}^{8}X_i=X$

So at least one X_i must be countably infinite.

Hence for at least one $1\le i\le 8$,

$\lim_{n\to\infty}\frac{|X_i(n)|}{n}=1$ and for all $1\le i\le n$

$\lim_{n\to\infty}\frac{|X_i(n)|}{n}\ge 0$

Q 118. Ans (2)

Q 119. Ans (1) (2) (3) (4)

Given I.E. is

$$u(x)=f(x)+\frac{2}{\pi}\int_0^{\pi}\sin(x-t)u(t)\,dt$$

It's Kernel $K(x,t)=\sin(x-t)$

$\Rightarrow$ $K(x,t)=\sin x\cos t-\cos x\sin t$

So, $f_1(x)=\sin x, g_1(t)=\cos t$

$f_2(x)=-\cos x, g_2(t)=\sin t$

Now the coefficient matrix for system $AX=B$ will have unique solution if eigenvalues of $A\ne\frac{1}{\lambda}=\frac{\pi}{2}$

where $A=\begin{bmatrix}\alpha_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\end{bmatrix}$

and where $\alpha_{ij}=\int_0^{\pi}g_i(x)f_j(x)dx$

$\Rightarrow$ $\alpha_{11}=\int_0^{\pi}\sin x\cos x\,dx=0$

$\alpha_{12}=\int_0^{\pi}-\cos^2 x\,dx=-2\int_0^{\pi/2}\cos^2 x\,dx$

$=-2\times\frac{1}{2}\times\frac{\pi}{2}=\frac{-\pi}{2}$

By walli's theorem

$\alpha_{21}=\int_0^{\pi}\sin^2 x\,dx=2\int_0^{\pi/2}\sin^2 x\,dx$

$=2\times\frac{1}{2}\times\frac{\pi}{2}=\frac{\pi}{2}$

$\alpha_{22}=\int_0^{\pi}-\cos x\sin x\;dx=0$

$\Rightarrow$ $A=\begin{bmatrix}0 & -\pi/2\\ \pi/2 & 0\end{bmatrix}$ which is real skew symmetric matrix whose eigenvalues are $\pm\frac{\pi}{2}$ (puerly imaginary). So for any value of $f(x)$ it has unique solution.

So all four options are correct.

Q 120. Ans (2) (3) (4)

Given that

R is non-zero ring with unity and $\forall r\in\mathbb{R}\;;r^2=r$

$\Rightarrow$ R is Boolean ring

$\Rightarrow$ $ch(R)=2$

If we take $R=\mathbb{Z}_2$ then R is an I.D.

so, option (1) is false

Now $(r+r)^2=r+r$

$\Rightarrow$ $r^2+r^2+r^2+r^2=r+r$

$\Rightarrow \quad r+r+r+r=r+r$

$\Rightarrow \quad r+r=0$

$\Rightarrow \quad r=-r$

$\Rightarrow$ Option (2) is correct.

Since Boolean ring is commutative

$\Rightarrow$ Option (4) is correct

And every non-zero prime ideal of Boolean ring is maximal.

[Non-trivial Boolean ring is either $\mathbb{Z}_2$ or extension of $\mathbb{Z}_2$].

So, options (2) , (3) & (4) are correct only.